Longitudinal elastic nonlinearity of composite material

B.N. Fedulov, D.A. Bondarchuk, E.V. Lomakin
Lomonosov Moscow State University, Russian Federation
fedulov.b@mail.ru, http://orcid.org/ /0000-0002-1894-5964
bondarchuk.da@gmail.com, http://orcid.org/0000-0001-7178-8585
evlomakin@yandex.ru, http://orcid.org/ 0000-0002-8716-5363

Citation: Fedulov, B.N., Bondarchuk, D.A., Lomakin, E.V., Longitudinal elastic nonlinearity of composite material, Frattura ed Integrità Strutturale, 67 (2024) 311-318.

Received: 07.11.2023
Accepted: 11.12.2023
Online first: 12.12.2023
Published: 01.01.2024

KEYWORDS. Fiber waviness, Continuum model, Elastic nonlinearity, Compressive modulus, Unidirectional composite.

INTRODUCTION

Composite materials have gained immense significance in a number of industrial sectors due to their exceptional mechanical properties and versatility. These materials typically consist of a combination of reinforcing fibers and a matrix, offer unique advantages such as high strength-to-weight ratios, corrosion resistance, and tailored performance characteristics. Understanding and optimizing the mechanical behavior of composite materials is of quite importance, particularly in industries where they play a critical role, such as aerospace, automotive, and construction. It is well known that the elastic properties of the composite depend on the type of loading [1-3]. For example, the difference in Young's modulus obtained under uniaxial tension and uniaxial compression in some laminated composites can reach up to 40%.

Elastic nonlinearity of composite material can be found for many moduli [4-6] and relations to model this effect were performed by many researchers [7-11]. For example, Hahn at al. [7] added fourth degree for shear stress component into complimentary energy to capture shear nonlinearity. Testa at al. [8] analyzed many stress components of an arbitrary degree in potential function to model fabric nonlinear deformations for biaxial loading. Yang at al. in their work [9] also used fourth degree polynomic function for stress components to describe complimentary energy for fabric material. Slovikov, S. V. et al. [10] introduced a linear decay of the stiffness during compression loading of longitudinal composites. Complex complimentary energy function with successful correlation was proposed by Obid at al. [11]. Their energy function was inspired by famous Ramberg-Osgood potential widely used for hyper-elastic materials.
Nevertheless, the question about longitudinal nonlinearity of unidirectional composite material is not completely unrevealed in terms of mechanism of appearance in uniaxial experiments and modelling approaches.
The idea to get properties for composite material using finite element modeling with RVE approach is promising. However, the use of conventional short periodicity cell does not predict stiffness difference in dependence of type of loading for longitudinal uniaxial case [12].
Analyzing nonlinearity of longitudinal stiffness of composite material one can found that to obtain accurate results, the defects, arising in the material during the manufacturing process, should be taken into account. Such a well-known defect as fiber waviness, can occur during the layout operations and curing process of the composite product. Waviness refers to the deviation of the fiber from rectilinearity. It is known, that fiber waviness significantly influences mechanical characteristics such as Young's modulus and compressive strength. A considerable amount of research is devoted to the problem of fiber waviness and its impact on composite properties [12-24].
This study aims to develop constitutive relations for the continuum model with taken into account longitudinal elastic nonlinearity of composite material. Second purpose of this research is to understand mechanism of this elastic nonlinearity of composite material. A special numerical experiment was conducted employing finite element modeling techniques. The simulation using large periodic cell with fiber waviness provides a good quality nonlinear loading curves which are used as input data for purposed continuum model. It was demonstrated the change in longitudinal stiffness of the AS4/8552 composite material under uniaxial compression in the fiber direction for subsequent waviness angles. The findings of this research hold the potential to provide valuable insights for engineering and materials science, with implications for the design and optimization of composite structures and components.

CONTINUUM MODEL

Modelling of composite material with several nonlinearity in chosen direction requires a change in the constitutive relations. Considering classical approach (1) in terms of complacencies with the extension of longitudinal coefficient A_{11} to be dependent on σ_{11}, makes us to get sureness that relations remain elastic. In other words we need to perform potential function for any dependence of $A_{11}(\sigma_{11})$.

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix} = \begin{bmatrix} A_{11}(\sigma_{11}) & A_{12} & A_{13} & 0 & 0 & 0 \\ A_{12} & A_{22} & A_{23} & 0 & 0 & 0 \\ A_{13} & A_{23} & A_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{23} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix} \tag{1}$$

Let us consider the complimentary energy in the following specific form:

$$\begin{aligned} \Phi = \frac{1}{2}((f(\sigma_{11})\sigma_{11} + A_{12}\sigma_{22} + A_{13}\sigma_{33})\sigma_{11} + (A_{12}\sigma_{11} + A_{22}\sigma_{22} + A_{23}\sigma_{33})\sigma_{22} + \\ +(A_{13}\sigma_{11} + A_{23}\sigma_{22} + A_{33}\sigma_{33})\sigma_{33} + \frac{\sigma_{12}{}^2}{G_{12}} + \frac{\sigma_{13}{}^2}{G_{13}} + \frac{\sigma_{23}{}^2}{G_{23}}) \end{aligned} \tag{2}$$

where $f\left(\sigma_{11}\right)$ – arbitrary function.

Taking derivatives one can obtain relations between stress components and deformations $\varepsilon_{ij}=\dfrac{\partial\Phi}{\partial\sigma_{ij}}$:

$$\varepsilon_{11}=\frac{\partial\Phi}{\partial\sigma_{11}}=\left(\frac{f'\left(\sigma_{11}\right)}{2}\sigma_{11}+f\left(\sigma_{11}\right)\right)\sigma_{11}+A_{12}\sigma_{22}+A_{13}\sigma_{33}$$

$$\varepsilon_{22}=\frac{\partial\Phi}{\partial\sigma_{22}}=A_{12}\sigma_{11}+A_{22}\sigma_{22}+A_{23}\sigma_{33}$$

$$\varepsilon_{33}=\frac{\partial\Phi}{\partial\sigma_{33}}=A_{13}\sigma_{11}+A_{23}\sigma_{22}+A_{33}\sigma_{33} \tag{3}$$

$$\gamma_{12}=\frac{\partial\Phi}{\partial\sigma_{12}}=\sigma_{12}/G_{12}$$

$$\gamma_{13}=\frac{\partial\Phi}{\partial\sigma_{13}}=\sigma_{13}/G_{13}$$

$$\gamma_{23}=\frac{\partial\Phi}{\partial\sigma_{23}}=\sigma_{23}/G_{23}$$

Eventually, we can assume:

$$A_{11}\left(\sigma_{11}\right)=\frac{f'\left(\sigma_{11}\right)}{2}\sigma_{11}+f\left(\sigma_{11}\right) \tag{4}$$

Ordinary differential Eqn. (4) always can be solved with initial conditions at $\sigma_{11}=0$,

$$\left.\frac{f'\left(\sigma_{11}\right)}{2}\sigma_{11}+f\left(\sigma_{11}\right)\right|_{\sigma_{11}=0}=A_{11}\left(0\right)=\frac{1}{E_1} \tag{5}$$

where E_1 is the elastic modulus in the vicinity of zero stress-strain values.

NUMERICAL EXPERIMENT

The numerical experiment is focused on investigating the impact of initial fiber waviness on the elastic modulus of the composite material in the fiber direction under uniaxial compression. The analysis takes into account the subsequent waviness angles: ψ=0°, 0.5°, 1.5°, and 3°. Fig. 1 illustrates the geometric method for determining the waviness angle.

The exact properties for fibers and matrix material to model AS4/8552 composite is taken in [12, 25, 26]. The choice of waviness angles for analysis motivated by the researches stated that the waviness angle for AS4/8552 composite material usually does not exceed 3° [26].

It was assumed, that the fiber has a sinusoidal shape. The properties of the fiber-matrix interface are neglected. Nonlinear behavior of the fiber and matrix materials was not considered in the modelling. Besides waviness, other defects were not taken into account. The lamina with curved (wavy) fibers is assumed to be flat. This assumption is typical for autoclave pre-preg, where waviness is primarily observed in the plane.

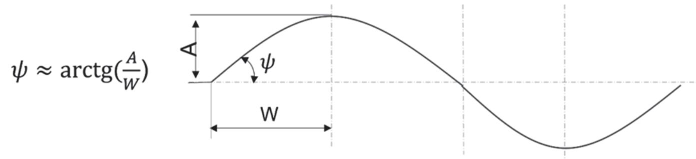

Figure 1: Fiber waviness.

Finite element model was developed in a three-dimensional formulation using a periodic cell. The periodic cell of the unidirectional composite material AS4/8552 with a fiber volume fraction of Vf=57% is shown in Fig. 2. The dimensions of the cell were chosen in such a way, that the volume fraction of 20 fibers corresponds to the fiber volume fraction in the composite material.

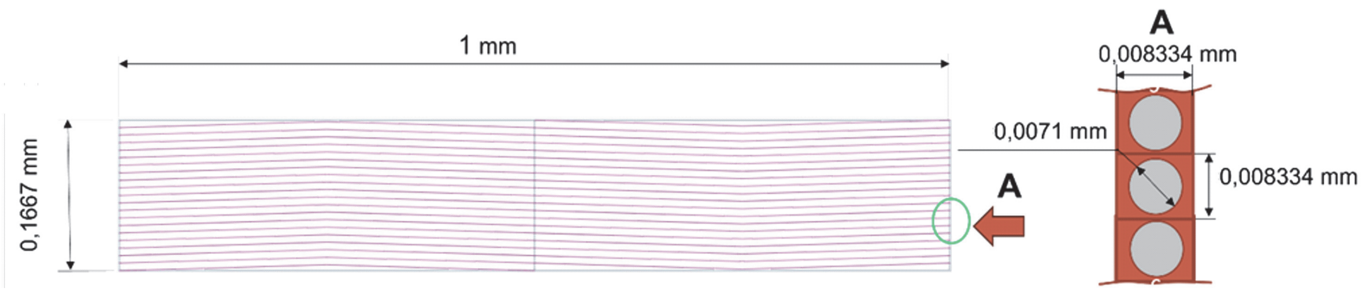

Figure 2: The periodic cell of the AS4/8552 composite material.

The finite element model of the periodic cell is presented in Fig. 3, with hidden center part of the matrix material.

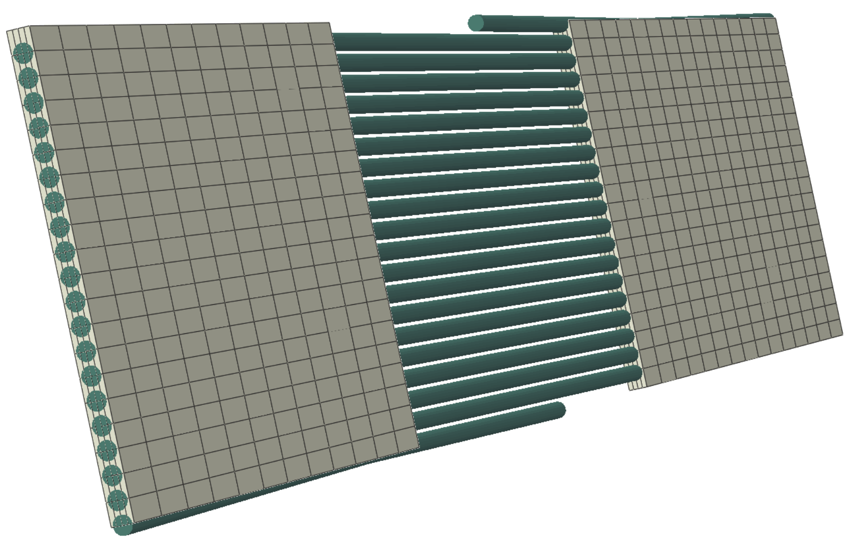

Figure 3: Finite element model.

The matrix was modeled using solid three-dimensional eight-node elements (C3D8I in ABAQUS notation), while the fibers were modeled using beam elements (B31 in ABAQUS notation). The interaction between the matrix and fibers was implemented using the embedded region constraint tool. The interface with no slip between the fiber and matrix is considered in this study. The fiber material was modeled as transversely isotropic linear elastic, while the matrix material was modeled as linear elastic isotropic material. No fracture or plasticity models were employed. The uniaxial compression of the unidirectional composite material realized by means of periodical boundary conditions. The geometrical nonlinearity was taken into account during performed analysis.

COMPUTATIONAL RESULTS

Fig. 4 shows stress-strain curves for uniaxial compression in the fiber direction of the AS4/8552 unidirectional composite material. The analysis includes the initial fiber waviness angles of 0°, 0.5°, 1.5° and 3°. Failure deformations for tension and compression ε_{ult}^{t} and ε_{ult}^{c}, shown in Fig. 4 taken from work [12].

Analysis of the modelling results shows continues reduction of longitudinal stiffness for specimens with built in waviness. Specimen without initial waviness performs constant stiffness without any smooth reduction. More over the results show that all characteristics provided by manufacturer [25, 26] correspond to the numerical specimen with 0.5° degree fiber waviness. Fig. 5 shows the results of compression loading simulation adopted in compliances to use for proposed continuum model.

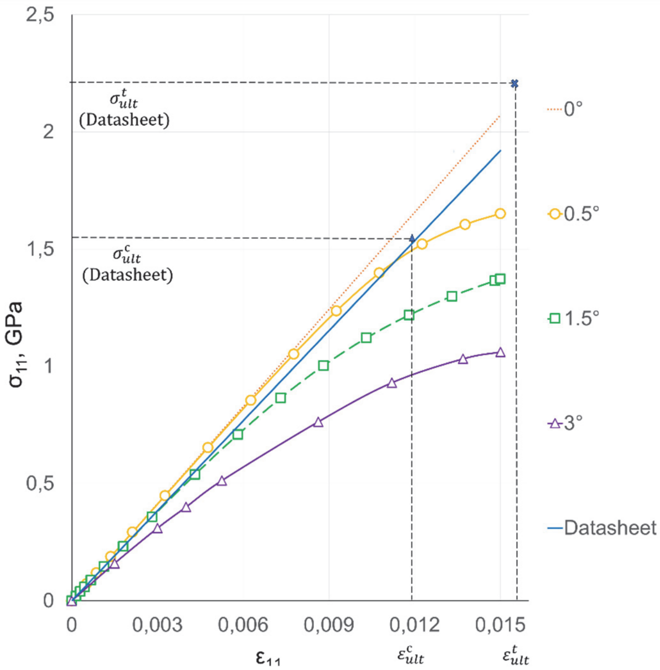

Figure 4: Numerically obtained stress-strain curve for uniaxial compression along the direction of reinforcement for unidirectional composite AS4/8552 with initial fiber waviness 0°, 0.5°, 1.5°, 3°.

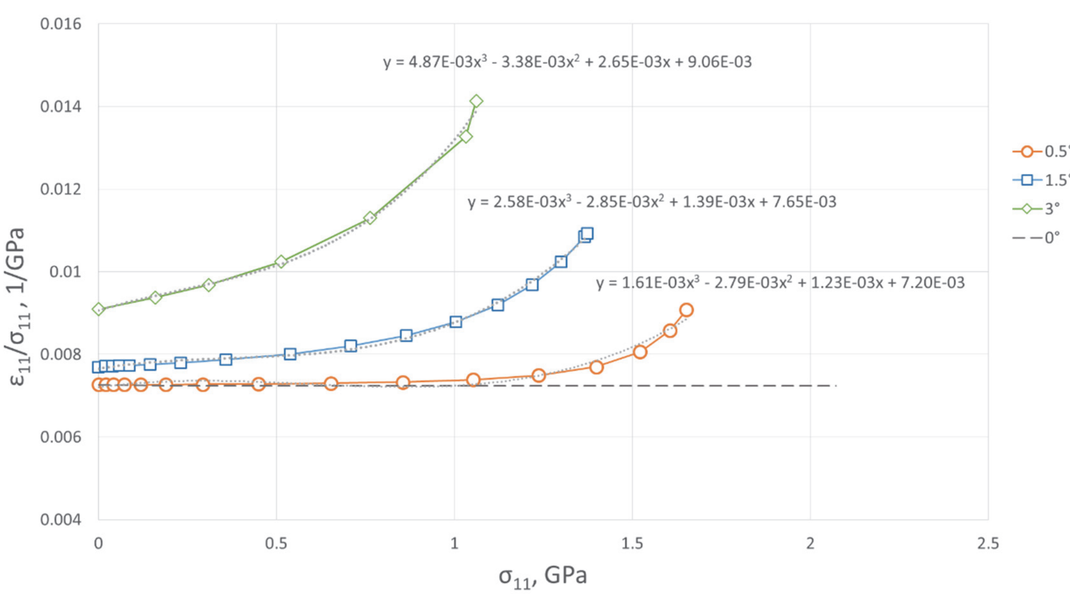

Figure 5: Numerically obtained compliance $\varepsilon_{11}/\sigma_{11}$ versus stress σ_{11} for uniaxial compression with initial fiber waviness 0°, 0.5°, 1.5°, 3°, for unidirectional composite AS4/8552.

Tab. 1 shows the coefficients of a polynomial approximation for $A_{11}(\sigma_{11}) = A\sigma_{11}{}^3 + B\sigma_{11}{}^2 + C\sigma_{11} + D$, obtained from numerical experiments with different initial fiber waviness.

Fiber waviness angle	A	B	C	D
0,5°	1.61E-03	−2.79E-03	1.23E-03	7.20E-03
1,5°	2.58E-03	-2.85E-03	1.39E-03	7.65E-03
3°	4.87E-03	−3.38E-03	2.65E-03	9.06E-03

Table 1: The coefficients for polynomic approximation $A_{11}(\sigma_{11}) = A\sigma_{11}{}^3 + B\sigma_{11}{}^2 + C\sigma_{11} + D$ of longitudinal compliances (1) for unidirectional composite AS4/8552 under uniaxial compression for initial waviness of 0.5°, 1.5° and 3°.

DISCUSSION

Solution of Eqn. (4) with approximation of $A_{11}(\sigma_{11})$ with cubic polynomic function obtained for potential function $f(\sigma_{11})$ can always be obtained as:

$$f(\sigma_{11}) = \frac{3A}{5}\sigma_{11}^3 + \frac{B}{2}\sigma_{11}^2 + \frac{2C}{3}\sigma_{11} + D \quad (6)$$

When analyzing work [1] where shear stiffness nonlinearity is modeled in the same manner, it can be seen that a solution for Eqn. (4) can be obtained for any polynomial function. This proves that longitudinal nonlinear compliance can be any arbitrary function, as meaningful experimental compliance can always be approximated by a polynomial function with any precision. Using the same approach it is possible to prove that arbitrary nonlinear compliance coefficients $A_{22}(\sigma_{22})$ and $A_{33}(\sigma_{33})$ keep constitutive relations (1) elastic. By combining this approach with the results obtained in work [1], we can conclude that all diagonal components can be dependent on the corresponding stress component with certainty of the elasticity of stress-strain relations. Additionally, the introduction of the triaxiality parameter [1] adds another layer of complexity to the model, allowing for different stiffness decay based on the type of loading. This level of detail and flexibility makes the model a valuable tool for analyzing and predicting the behavior of composite materials under various loading conditions.

CONCLUSION

In this study, the continuum model taking into account the nonlinearity of the longitudinal stiffness of the composite was developed. The mathematical proof of the potentiality of the proposed relations was performed. It was shown, that elastic longitudinal compliance coefficient could be an arbitrary dependence of longitudinal stress component until they can be approximated by polynomial function.

Based on the modeling results using the periodicity cell approach, it was observed, that the considering of geometric nonlinearity in combination with initial fiber waviness gives a decrease of stiffness on the compression stress-strain diagram. However, this effect is not observed during tension. The modeling shows that the stiffness does not change abruptly. There is a smooth decay in stiffness during compression as the strain increases. This decrease occurs only for specimens with initial fiber waviness.

ACKNOWLEDGEMENTS

This work was supported by the Russian Science Foundation, grant No. 20-11-20230-P.

REFERENCES

[1] Lomakin, E. V. and Fedulov, B. N. (2015). Nonlinear anisotropic elasticity for laminate composites. Meccanica, 50, pp. 1527-1535. DOI https://DOI.org/10.1007/s11012-015-0104-5

[2] Alves, M.P., Cimini Junior, C.A., Ha, S.K. (2021). Fiber waviness and its effect on the mechanical performance of fiber reinforced polymer composites: An enhanced review, Compos Part A Appl Sci Manuf, 149, p. 106526. DOI: 10.1016/j.compositesa.2021.106526.

[3] Hahn, H. T. and Williams, J. G. (1986). Compression failure mechanisms in unidirectional composites, pp. 115-139. West Conshohocken, PA, USA: ASTM International.

[4] Puck, A. and Schürmann, H. (2004). Failure analysis of FRP laminates by means of physically based phenomenological models (1996), in Failure Criteria in Fibre-reinforced-Polymer Composite, pp. 832-876. DOI: 10.1016/B978-008044475-8/50028-7

[5] Smith, E. W. and Pascoe, K. J. (1977). The role of shear deformation in the fatigue failure of a glass fibre-reinforced composite. Composites, 8(4), pp. 237-243. DOI: 10.1016/0010-4361(77)90109-4

[6] Van Vuure, Aart W., (2015). Compressive properties of natural fibre composites." Materials Letters 149 , pp. 138-140.

[7] Hahn, H. T. and Tsai, S. W. (1973). Nonlinear Elastic Behavior of Unidirectional Composite Laminae. Journal of Composite Materials, 7(1), pp. 102–118. DOI: 10.1177/002199837300700108

[8] Testa, R. B. and Yu, L. M. (1987). Stress-strain relation for coated fabrics. Journal of engineering mechanics, 113(11), pp. 1631-1646. DOI: 10.1061/(ASCE)0733-9399(1987)113:11(1631

[9] Yang, B., Yu, Z., Zhang, Q., Shang, Y. and Yan, Y. (2020). The nonlinear orthotropic material model describing biaxial tensile behavior of PVC coated fabrics. Composite Structures, 236, 111850. DOI: 10.1016/j.compstruct.2019.111850

[10] Slovikov, S. V., Babushkin, A. V. and Gusina, M. D. (2023). Nonlinearity of compression behavior of 3D-epoxy reinforced with carbon fibers composites. Frattura ed Integrità Strutturale 66, pp. 311-321. DOI: 10.3221/IGF-ESIS.66.19.

[11] Obid, Š., Halilovič, M., Urevc, J. and Starman, B. (2023). Non-linear elastic tension–compression asymmetric anisotropic model for fibre-reinforced composite materials. International Journal of Engineering Science, 185, 103829. DOI: 10.1016/j.ijengsci.2023.103829

[12] Naya, F., Herráez, M., Lopes, C.S., González, C., Van der Veen, S., Pons, F. (2017). Computational micromechanics of fiber kinking in unidirectional FRP under different environmental conditions, Compos Sci Technol, 144, pp. 26–35. DOI: 10.1016/j.compscitech.2017.03.014.

[13] Budiansky, B., Fleck, N.A. (1993). Compressive failure of fibre composites, J Mech Phys Solids, 41, pp. 183–211. DOI: 10.1016/0022-5096(93)90068-Q

[14] Curtis, G.J., Milne, J.M., Reynolds, W.N. (1968). Non-Hookean Behaviour of Strong Carbon Fibres, Nature, 220 (5171), pp. 1024–1025. DOI: 10.1038/2201024a0.

[15] Drummer, J., Tafesh, F., Fiedler, B. (2023). Effect of Fiber Misalignment and Environmental Temperature on the Compressive Behavior of Fiber Composites, Polymers (Basel), 15(13), p. 2833. DOI: 10.3390/polym15132833.

[16] Herraez, M., Bergan, A., Gonzȧlez, C., Lopes, C. s. (2018). Modeling Fiber Kinking at the Microscale and Mesoscale.

[17] Herráez, M., Bergan, A.C., Lopes, C.S., González, C. (2020). Computational micromechanics model for the analysis of fiber kinking in unidirectional fiber-reinforced polymers, Mechanics of Materials, 142, p. 103299. DOI: 10.1016/j.mechmat.2019.103299.

[18] Hsiao, H.-M., Daniel, I.M. (1996). Effect of fiber waviness on stiffness and strength reduction of unidirectional composites under compressive loading, Compos Sci Technol, 56, pp. 581–593. DOI: 10.1016/0266-3538(96)00045-0

[19] Keryvin, V., Marchandise, A., Grandidier, J.-C. (2022). Non-linear elastic longitudinal behaviour of continuous carbon fibres/epoxy matrix composite laminae: Material or geometrical feature?, Compos B Eng, 247, p. 110329. DOI: 10.1016/j.compositesb.2022.110329.

[20] Meng, M., Le, H.R., Rizvi, M.J., Grove, S.M. (2015). The effects of unequal compressive/tensile moduli of composites, Compos Struct, 126, pp. 207–215. DOI: 10.1016/j.compstruct.2015.02.064.

[21] Mrse, A.M., Piggott, M.R. (1993). Compressive properties of unidirectional carbon fibre laminates: I. A compact flexure beam for compression testing, Compos Sci Technol, 46(3), pp. 213–217. DOI: 10.1016/0266-3538(93)90155-A.

[22] Safdar, N., Daum, B., Rolfes, R. (2022). The effect of model dimensionality on compression strength of fiber reinforced composites, J Compos Mater, 56(30), pp. 4645–4662. DOI: 10.1177/00219983221136272.

[23] Zeng, W.H., He, P. and Chen, G.F. (2017). The effects of fiber waviness on the stiffness of composite laminates. ICCM-21, Xi'an, China.

[24] Stewart, A.L., Poursartip, A. (2018). Characterization of fibre alignment in as-received aerospace grade unidirectional prepreg, Compos Part A Appl Sci Manuf, 112, pp. 239–249. DOI: 10.1016/j.compositesa.2018.04.018.
[25] Product data HexPly® 8552. Available at: https://energy.ornl.gov/CFCrush/materials/uou/8552_eu.pdf.
[26] Clarkson, E. (2011). Hexcel 8552 AS4 unidirectional prepreg qualification statistical analysis report. FAA special project no SP4614WI-Q, Report No NCP-RP-2010-008 Rev D, Wichita State University. Available at: https://www.wichita.edu/industry_and_defense/NIAR/Research/hexcel-8552/AS4-Unitape-3.pdf