



## A simplified formula to estimate the load history due to ballistic impacts with bullet splash. Development and validation for finite element simulation of 9x21mm full metal jacket bullets

Riccardo Andreotti

*Callens® AREA3, Via Merini 37 21100 Varese, Italy*

*Dipartimento di Meccanica, Politecnico di Milano, via La Masa 1, 20156 Milano, Italy*

*riccardo.andreotti@callens.it*

Andrea Casaroli

*Dipartimento di Meccanica, Politecnico di Milano, via La Masa 1, 20156 Milano, Italy*

*andrea.casaroli@polimi.it*

Mauro Quercia

*Callens® AREA3, Via Merini 37 21100 Varese, Italy*

*mauro.quercia@area3.it*

Marco V. Boniardi

*Dipartimento di Meccanica, Politecnico di Milano, via La Masa 1, 20156 Milano, Italy*

*marco.boniardi@polimi.it*

**ABSTRACT.** An original simplified formula is proposed to estimate the load history caused by ballistic impacts characterized by the so-called bullet splash phenomenon, consisting in the complete bullet fragmentation with no penetration of the target. The formula is based on the progressive momentum variation of the mass of the bullet impacting on a planar plate normal to the impact direction. The method aims at creating a simplified approach to assess the response of structures by means of explicit finite element simulations without the need of modelling the interaction between impactor and target. The results demonstrate that the proposed method can be used to estimate the forces generated by bullet-splash phenomena of 9x21mm full metal jacket bullets and effectively applied to finite element simulations allowing significant reductions in computational cost.

**KEYWORDS.** Ballistic impact; Load history; Bullet splash; Stainless steel plate; Finite element simulation (FEM); Explicit solver.



**Citation:** Andreotti R., Casaroli A., Quercia M., Boniardi M.V., A simplified formula to estimate the load history due to ballistic impacts with bullet splash. Development and validation for finite element simulation of 9x21mm full metal jacket bullets, 62 (2022) 602-612.

**Received:** 12.08.2022

**Accepted:** 14.09.2022

**Online first:** 15.09.2022

**Published:** 01.10.2022

**Copyright:** © 2022 This is an open access article under the terms of the CC-BY 4.0, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.



## INTRODUCTION

In today's aerospace and defense industry, the optimization of protective capabilities of a structural system is mainly done by means of finite element explicit simulations, with a similar approach to what is done for passive safety requirements like bird strike [1]. This paper aims at proposing and discussing the effectiveness of an easy-to-use formula to estimate the evolution in time of the impactor-target interaction forces due to bullet splash [2], which represent a typical load case of a properly working protective structure able to locally withstand the interaction with the impactor, therefore generating high intensity reaction forces that propagate to the surrounding structures threatening their general integrity. The estimation of the load history allows analysts to conduct finite element simulations of bullet splash phenomena to predict the response of protective structural systems with the significant advantage of avoiding the cost of modelling and simulating the interaction phenomena between impactor and target. A similar approach was followed by the U.S. ARMY ballistic research laboratory of Aberdeen, Maryland with the development of the EPIC-2 code [3], but the code is not available.

The available literature mostly proposes simulation techniques for ballistic impacts focusing on the effects of rigid projectiles on deformable targets, to predict the ballistic limit velocity and the failure modes of the target. In 2021 Goda et al. [4] analyzed the damage and stress field of a modern ceramic/composite protection impacted and penetrated by rigid projectiles. In 2022 the same author focused on the effects of the shape and angle of impact on the energy absorption of woven-fabric protections again impacted and penetrated by rigid projectiles [5]. Yunfei et al. [6] (2014) verified both numerically and experimentally the effects of impactors of different strengths and deformability on penetrating metallic plates. Rajput & Iqbal (2017) [7] instead showed the effects of the nose shapes of rigid projectiles impacting on aluminum plates at different velocity. Regarding the effects of the fragmentation of the bullet, Bresciani et al. [8] in 2016 published a study about the use of adaptive remeshing and smooth particle hydrodynamics (SPH) methods to investigate the interactions between impactor and target when they both encounter fragmentation.

In this context, Andreotti et al. (2021) [9] proposed a simplified finite element approach to simulate the interaction between bullet fragments and target during bullet splash phenomena. The model then proposed was based on an arbitrary lagrangian-eulerian (ALE) formulation to simulate the interaction between fragments and target as a fluid structure interaction (FSI), therefore avoiding the mechanical characterization of the deviatoric components of the constitutive law assigned to the bullet's material. The approach was experimentally validated for 4 mm thick plates made with AISI 304 steel hit by 9x21mm full metal jacket bullets both in terms of back plate residual displacements and in terms of plastic strain field in the impact area.

The work here presented was conceived to take advantage of that experience and propose an even more efficient way to face the problem of assessing protective structural systems against bullet splash phenomena once the focus of the simulations to be carried out is to investigate the response of the structures in the surroundings of the impact point without the need of a detailed reproduction of the local strain field due to the direct pressure of the fragments hitting the epicenter of the impact. This paper presents a formula to estimate the resultant load history due to bullet splash interactions, based only on the geometry of the bullet, its mass distribution, and its initial impact velocity. The formula is aimed to be useful to define load curves to be applied to simplified finite element models to conduct explicit dynamics analyses on any simulation platform featured with an explicit structural solver.

The approach has been validated by comparing the results of the detailed fluid-structure interaction already experimentally validated by Andreotti et al [9] with the results of a simplified simulation in which the load history is estimated by means of an analytical-numerical approach based on the proposed theoretical formula.

The comparisons between the history of global reaction forces and the stress waves being transferred through the plates during the simulations show a clear consistency of the method. Moreover, the comparison between the FSI and non-FSI calculation times shows a clear advantage in terms of computational cost.

Section 2 of the article illustrates the development of the load history formula and its practical application to the 9x21 FMJ bullet considered by Andreotti et al. [9]. Section 3 explains how the finite element simulations were conducted. In section 4 the results are discussed comparing them with the ones obtained by means of the FSI model. Section 5 summarizes the results and conclusions introducing possible further developments to the research.

## ESTIMATION OF THE LOAD HISTORY

According to Andreotti et al. [9], bullet splash phenomena are mainly governed by the inertial, geometrical and compressibility properties of the impactor, while the deviatoric part of the constitutive law of the bullet's materials can be considered neglectable. The simplification effectively introduced to treat bullet splash as a fluid structure



interaction, where the fluid represents the mass of the bullet's debris flowing against the target's surface, suggests therefore a further simplification, valid for splashing bullets hitting planar surfaces at 90° incidence angle, which is the worst impact angle a ballistic protection can face. This simplification consists in estimating the load generated by the interaction between bullet's debris and target as the resultant force needed to progressively deviate the trajectory of the flux of bullet's material, considering the displacements of the target as neglectable. This allows to decouple the fluid-structure interaction and treat the problem as a transient phenomenon during which the structure is loaded by an already known load history, therefore avoiding the computational cost due to modelling the FSI.

The resultant force  $F(t)$  to be applied in the impact direction  $x$  is estimated as the time derivative of the momentum of the bullet fragments under the hypothesis that the only effect of the impact on bullet's material is a 90-degree deflection of its trajectory, gradually happening during the relative movement of the bullet with respect to the target, considered rigid and fixed (Fig. 1).

To calculate the load history, let's consider a generic time  $t$  after the first contact between impactor and target and consider the elementary variation of the momentum  $dq$ , happening from time  $t$  to time  $t+dt$ , that would be

$$dq_x = dm(v_{xi} - v_{xf}) \tag{1}$$

where  $dm$  is the mass of debris deflected from time  $t$  to  $t+dt$  while  $v_{xi}$  and  $v_{xf}$  are the  $x$  components of the velocity of the debris before and after the deflection.

As a result of the 90-degree deflection hypothesis  $v_{xf}$  can be considered null, and considering only 90-degree impacts  $v_{xi}$  is equal to the initial impactor velocity  $v$ , therefore Eqn. (1) becomes:

$$dq = dm v \tag{2}$$

In the hypothesis that the bullet is homogeneous, and no perturbations occur to the bullet's particles until they ideally intersect the target's surface, the elementary mass  $dm$  can be expressed as:

$$dm = \rho A(t) v dt \tag{3}$$

where  $A(t)$  represents the area of intersection between the bullet's undeformed volume and the plane lying on the impact surface of the target at time  $t$ ,  $\rho$  is the density of the associated material and  $v dt$  represents the elementary displacement  $ds$  describing the kinematics of the unperturbed part of the bullet from time  $t$  do  $t+dt$ :

$$ds = v dt \tag{4}$$

We can now substitute Eqn. (3) into Eqn. (2) and divide both terms for  $dt$ , obtaining the estimation of the impact force  $F$  at time  $t$ , as a function of initial velocity, inertia, and geometry of the impactor:

$$F(t) = \frac{dq(t)}{dt} = \rho A(t) v^2 \tag{5}$$

By integrating in time Eqn. (5) we obtain the definition of the initial momentum  $q$  of the impactor:

$$q = \int F(t) dt = \rho v \int A(t) v dt = \rho v \int A(s) ds = \rho v V = v M \tag{6}$$

where  $V$  is the volume of the homogeneous impactor and  $M$  is its total mass. However, most bullets are not homogeneous, so the hypothesis of homogeneity must be overcome to apply the formula to real world problems. We can easily generalize the formula to consider heterogeneous bullet's sections by introducing the resultant impact force as the sum of  $m$  contributions:

$$F(t) = \sum_{i=1}^m F_i(t) \tag{7}$$



where  $m$  is the number of different materials composing the impactor and  $F_i(t)$  is the force contribution due to the  $i$ -material at time  $t$ .

Therefore, the load history due to the bullet splash of an impactor whose generic section is composed by  $m$  different materials can be expressed as:

$$F(t) = v^2 \sum_{i=1}^m \rho_i A_i(t) \tag{8}$$

where  $\rho_i$  is the density of the  $i$ -material, and  $A_i(t)$  is the area of the  $i$ -material ideally intersecting the surface of the target at time  $t$ .

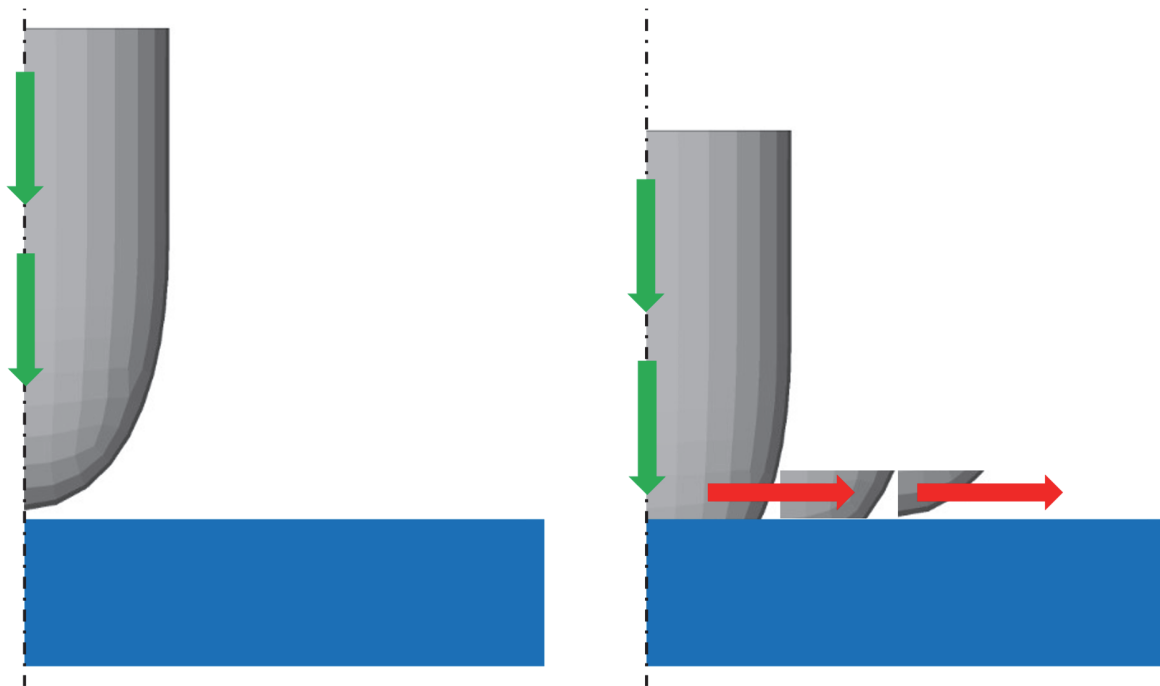


Figure 1: Schematic simplification of the 90-degree bullet splash. The deflection of the debris is a continuous process during which the portion of the bullet ideally intersecting the target's surface is perfectly deflected in radial direction and no perturbation occurs to the portion of the bullet not yet having intersected the surface of the target.

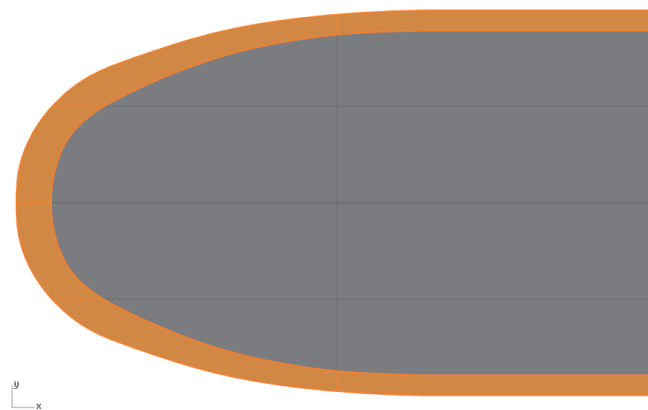


Figure 2: Graphic representation of the bullet's section composed by the brass jacket (orange) and the lead filler (gray).

*Application of the method to estimate the load history due to 9x21mm FMJ bullet splash at 322m/s*

A concrete application of the proposed method consists in analyzing the section of the bullet (Fig. 2 and Fig. 3) to identify the material distribution along its cross section to build the  $A_i(t)$  functions (Fig. 4), and then applying Eqn. (8) to calculate

the resultant load history (Fig. 5). Considering  $\rho = 10750 \text{ kg/m}^3$  as the density of the lead-based alloy filler and  $\rho = 8730 \text{ kg/m}^3$  as the density of the brass jacket, the process leads to the calculation of the load history, giving as a result a force history that grows from null to 68430 N in around 0.03s and keeps that intensity until  $t=0.0466\text{s}$  when the estimated interaction phenomenon ends.

As a control, the integration in time of the estimated total load history  $F(t)$  (Fig. 5) gives a total variation of the bullet's momentum equal to 2.576 kgm/s, which correctly corresponds to the product of the impact velocity (322 m/s) multiplied by the nominal mass of the 9x21mm FMJ bullet (8 g).

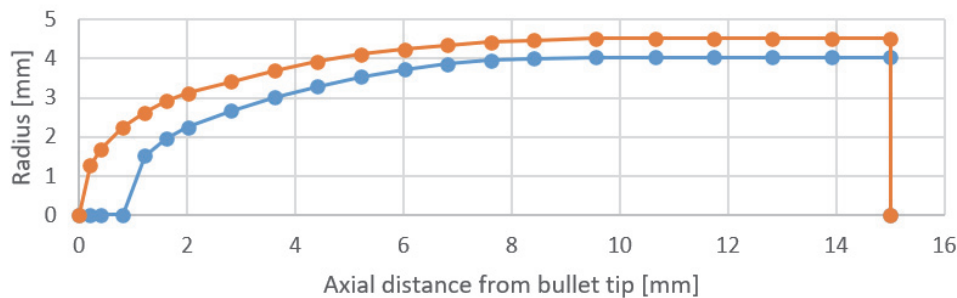


Figure 3: Measured radius values (ordinate axis in mm) of the boundary surfaces of the materials all along the axial coordinate of the bullet (abscissa in mm).

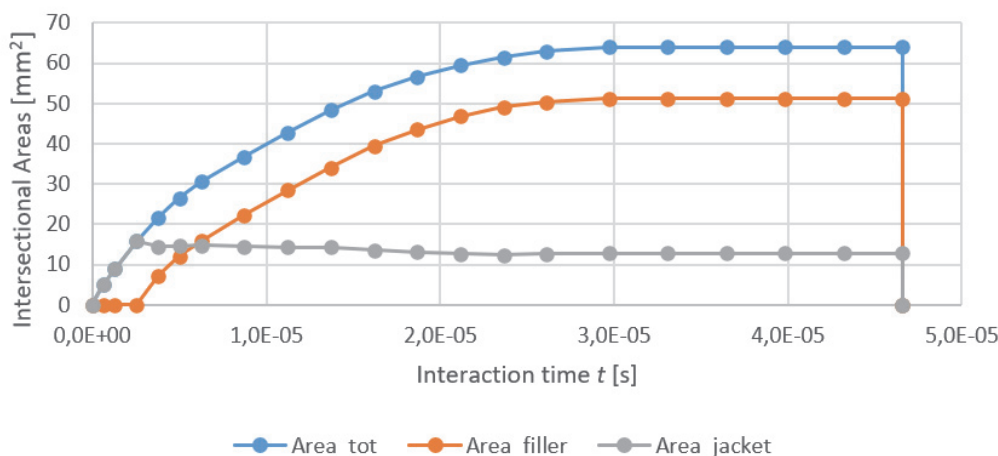


Figure 4: Values of the intersectional areas  $A_i$  [mm<sup>2</sup>] (ordinate) as functions of interaction time  $t$  [s] (abscissa) and their sum.

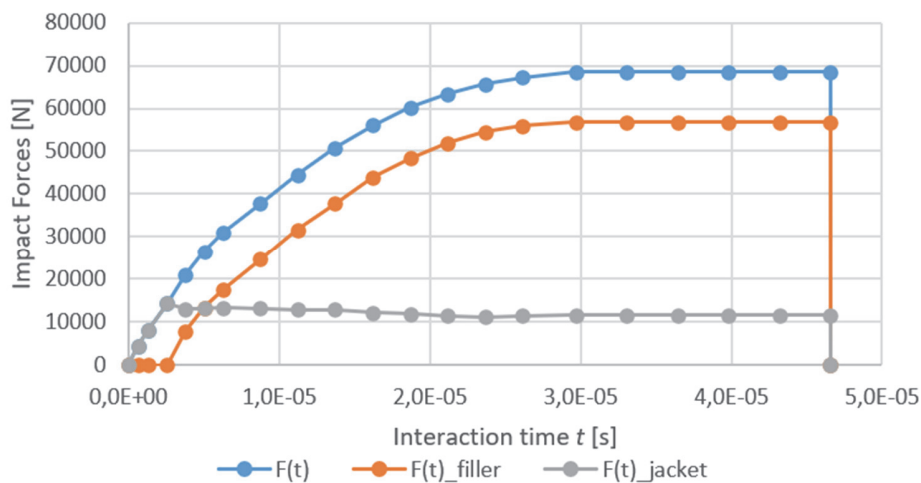


Figure 5: Values of the estimated impact forces  $F_i$  [N] (ordinate) as functions of interaction time  $t$  [s] (abscissa). The total value of the impact force is represented in blue, the component due to the lead filler is represented in red, the component due to the brass jacket is represented in gray.

## FINITE ELEMENT SIMULATIONS

To validate the load history approach we simulated the same impact analyzed by Andreotti et al. [9] to validate the FSI method. The simulated impact is therefore a 9x21mm full metal jacket (FMJ) bullet hitting a 250x250mm 4 mm thick AISI 304L plate at 322 m/s with an impact angle of 90degree.

### *Geometrical discretization of the plate*

To allow proper comparison between FSI approach and load history approach, a first round of simulations was performed with the same structural finite element model used by Andreotti et al. [9], with a squared 60x60 mm area of the plate around the impact point discretized in 8-nodes solid elements with 0.2 mm size. The remaining part of the plate was instead simplified with 2.5mm shell fully integrated 4-nodes elements connected to the solids.

A double symmetry plane boundary condition is associated with the model. The displacements of the external boundary nodes of the shell plate are constrained in the impact direction (Fig. 6).

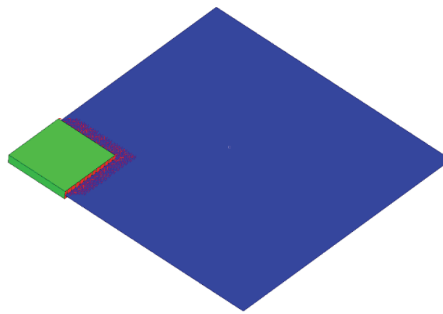


Figure 6: Geometrical discretization of the plate: shell elements (blue), solid elements (green).

### *Mechanical characterization of the plate*

To allow proper comparisons, the static and dynamic constitutive model of the AISI 304L associated with the plate was taken from Andreotti et al. [9].

### *Comparative simulations*

Four simulations have been conducted. The first simulation (A) is a repetition of the FSI simulation based on the Arbitrary-Lagrangian-Eulerian method (ALE) adopted by Andreotti et al. [9]. The second simulation (B) is the application of the estimated load history as a distributed load acting on the epicenter as a uniform pressure applied on a circular area equal to the nominal cross section of the bullet, i.e. with 4.5mm radius. The third simulation (C) is again the application of the estimated load history as a uniform distributed load acting on a circular area, this time with a radius increased by 50% to take into account the real interaction area as experimentally analyzed by Andreotti et al. [9]. At last, some fourth and fifth simulations (D and E) were conducted on a full-shell plate model loaded with the estimated load history again distributed on a circular area with respectively 4.5mm and 6.75mm radiuses. Tab. 1 summarizes all the features of the conducted simulations. All the numerical simulations were conducted by means of the explicit solver LSDYNA [10].

Simulation	Loading method	Pressure distribution	FE model of the plate
A	FSI [9]	Variable Pressure Field (ALE) [9]	Solid (0.2mm) – Shell (2.5mm)
B	Estimated load history	Uniform Circular (4.5mm radius)	Solid (0.2mm) – Shell (2.5mm)
C	Estimated load history	Uniform Circular (6.75mm radius)	Solid (0.2mm) – Shell (2.5mm)
D	Estimated load history	Uniform Circular (4.5mm radius)	Shell (2.5mm)
E	Estimated load history	Uniform Circular (6.75mm radius)	Shell (2.5mm)

Table 1: Summary of the simulations conducted.



## RESULTS AND VALIDATION

To evaluate how well the proposed load history method could effectively substitute the FSI approach for the purpose of assessing the response of a structural system impacted by ballistic impacts with bullet splash, in the following we compare the results of the simulations in terms of resultant forces at the constraints, local stress waves transmitted from the epicenter to the periphery of the plate and normalized computational cost of each simulation. Having the FSI approach been experimentally validated by Andreotti et al. [9], the results of simulation A will be treated as a benchmark for the proposed alternative method. Further considerations will be carried out by comparing the predicted back-plate residual deformations of the plate with the experimental data.

### *Total reaction forces*

The comparison between the histories of the total reaction force needed to contrast the impact along the boundary of the plate shows very good adhesion of the results, with the peaks corresponding to the first back-and-forth bounces being very similar both in amplitude and phase. No significant spread is visible between benchmark simulation A and test simulations B and C, with less than 10% difference in peaks amplitude, showing substantial equivalence between the FSI method and the proposed estimated-load-history method in terms of global reaction forces. Full-shell simulations D and E show slightly shorter oscillation periods in the bounces happening at the end of the simulation time; the difference in phase is evident after 0.3ms and is due to the slightly higher stiffness resulting from the reduction in degrees of freedom due to the adoption of a looser shell mesh. Moreover, the comparison between B and C as well as between D and E shows the invariance of the global forces with respect to the arbitrarily chosen pressure distribution areas.

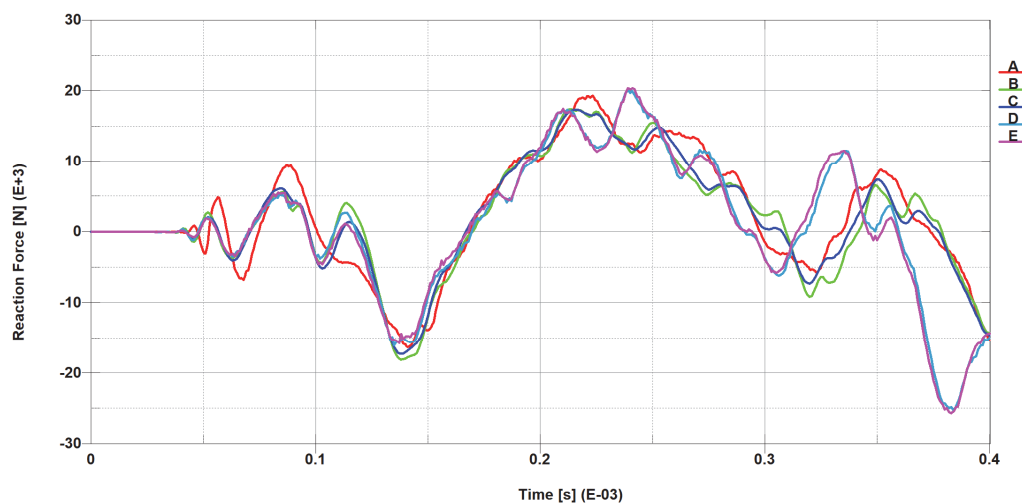


Figure 7: Comparison between the total reaction forces in the impact direction, showing good adhesion of the results.

### *Stress waves propagation*

The propagating stress waves generated by the simulated impacts are very similar. As displayed in Fig. 11, the amplitude of the stress waves propagating from the epicenter at the end of the interaction time is very similar across all the simulations. To compare in more detail those results, we chose two points at half-way radial distance from epicenter to the constrained periphery of the plate (Fig. 8). Point 1 stays on the diagonal of the plate (maximum radial distance from epicenter to the constrained nodes). Point 2 stays on the cross section of the plate where the radial distance from the loaded point to the fixed nodes at the boundary is minimum. In general, the comparison between the histories of maximum principal stress shows good adhesion of the stress waves. At Point 1 (Fig. 9) simulations B and C underestimate the peak stress by around 20% with respect to A. Full-shell simulations D and E also slightly underestimate the peak stress of about 8%. At point 2 (Fig. 10) simulations B and C underestimate the peak stress of around 5%; simulations D and E, instead, overestimate the peak stress of about 10%. No significant differences are visible between simulations B and C, and no significant differences are visible between simulations D and E, again confirming that the arbitrary extension of the loaded area and the local intensity of the pressure field is non relevant within the tested range.



As already observed on the reaction forces, full shell simulations D and E show slightly shorter oscillation periods in the bounces happening at the end of the simulation time. The difference in phase and amplitude is evident after 0.3ms and is due to the slightly higher stiffness due to the less degrees of freedom characterizing the looser shell mesh adopted.

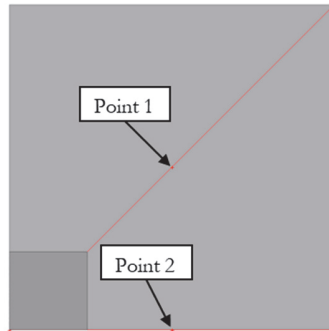


Figure 8: Points of comparison between the stress waves.

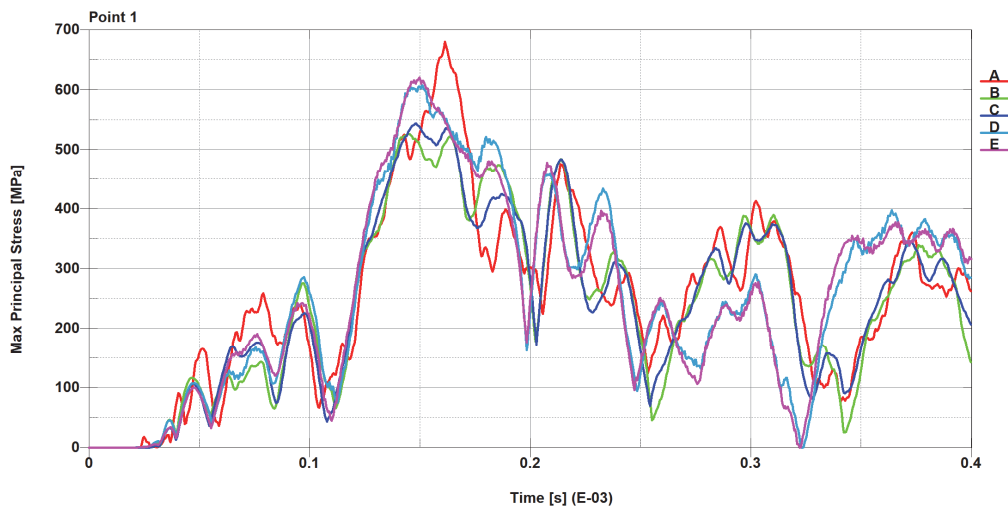


Figure 9: Comparison between the stress waves in terms of maximum principal stress at point 1.

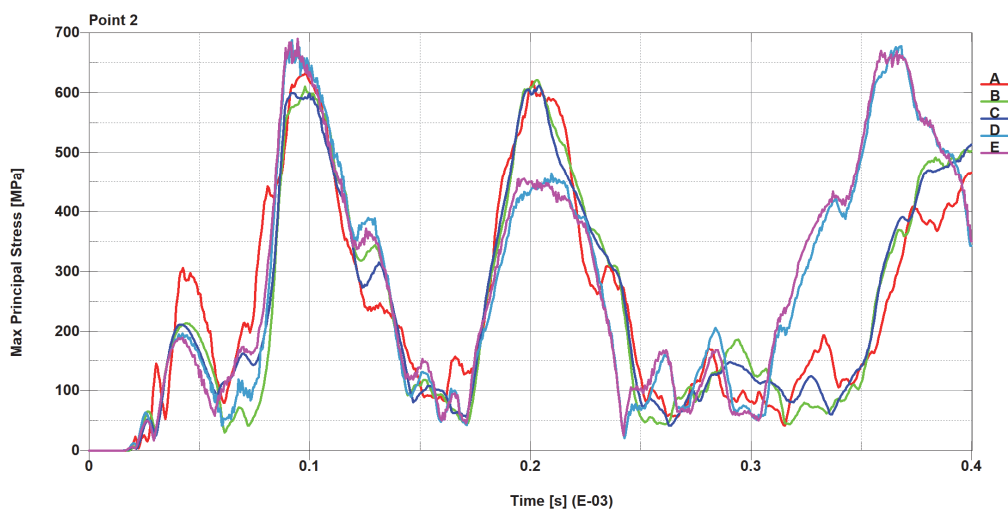


Figure 10: Comparison between the stress waves in terms of maximum principal stress at point 2.



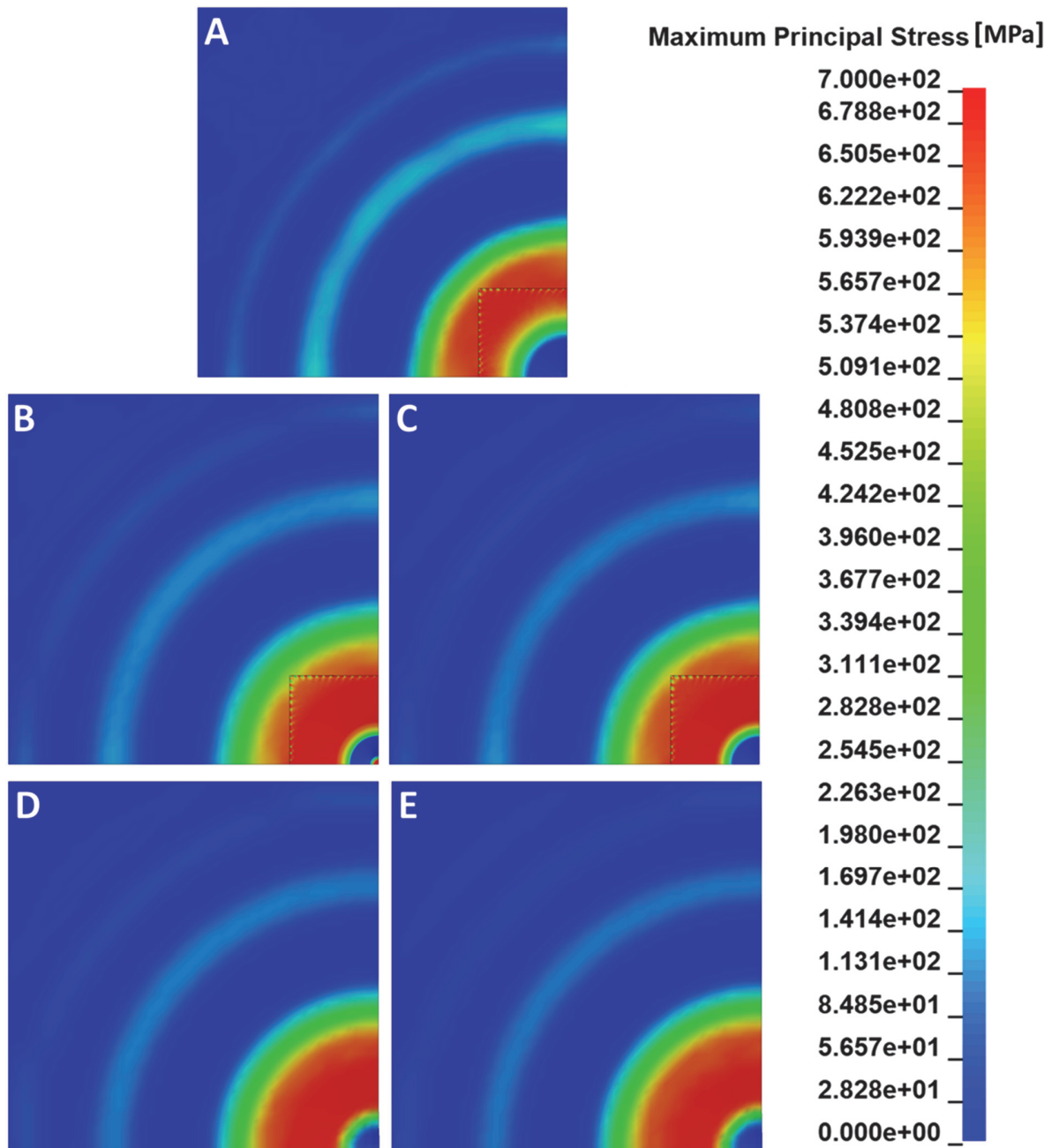


Figure 11: Comparison between the stress waves in terms of maximum principal stress at 0.05ms (right after the end of the interaction time). On first row simulation A, on second row simulations B and C (left to right), on third row simulations D and E (left to right). The stress scale is in MPa.

### *Residual displacements*

Compared to the experimental data and the A simulation (Fig. 12) the residual displacements predicted by the simulations B, C, D and E are overestimated between +56% (simulation B) and +15% (simulation E). It is evident, however, how the radius of the chosen circular area on which the uniform pressure was arbitrarily applied has a very significant impact on residual displacements, with simulations C and D (6.75mm radius) giving much better predictions than B and D (4.5mm radius). Even though local effects are out of the scope of this study, this observation suggests the possibility to even identify an *equivalent radius* that could better predict the residual back plate deformation.

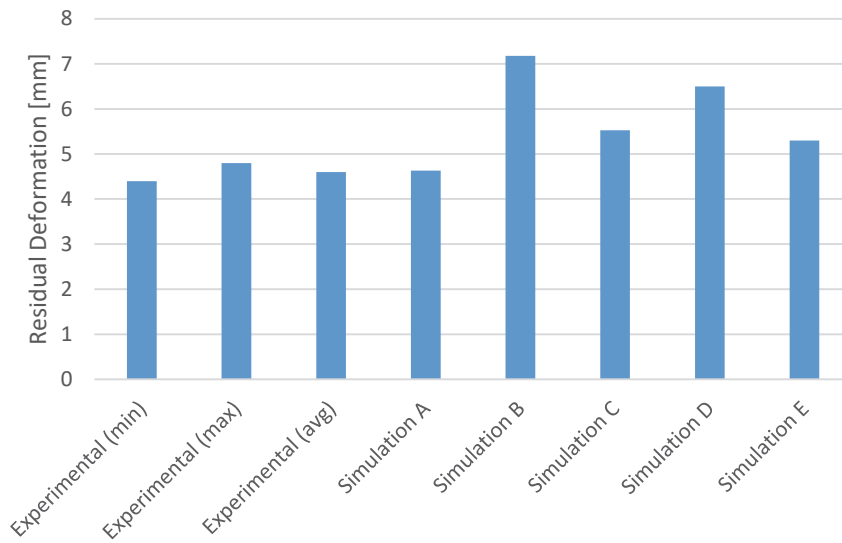


Figure 12: Comparison between residual displacements as measured experimentally (from left: the minimum, maximum and average values) and simulations results.

#### Calculation times

Compared to simulation A, the elimination of the FSI allowed simulations B and C to finish in around 45% the amount of calculation time. Furthermore, the substitution of shell elements in place of the detailed 3D solid mesh from, needed at the epicenter to conduct detailed investigation using FSI, allowed simulations D and E to run enormously faster, in less than 0.2% of the time needed by simulation A (Fig. 13). The comparison was obviously conducted at equal conditions.

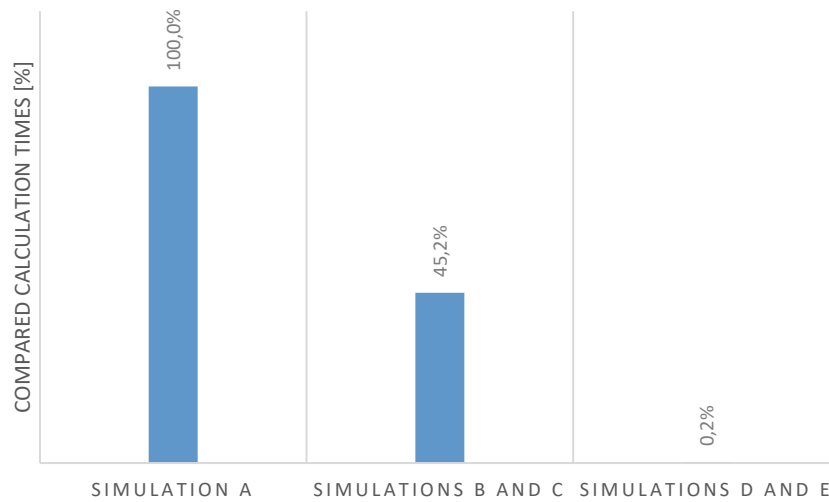


Figure 13: Comparison between the calculation times needed to obtained the above results. The histogram represents the percentage ratios between the calculation's times needed by the simulations divided by the calculation time needed by our benchmark simulation A. Simulations B, C and D, E are represented in the same column because their calculation times are equal due to the same structural model used (see Tab. 1).



## CONCLUSIONS

The aim of this study was to identify and validate a more efficient way to assess the global response of a structural system to ballistic impacts with bullet splash. The method proposed is based on the ideal continuous fragmentation of the bullet. The impact forces are calculated as the time derivative of the momentum of the flux of fragments.

The proposed formula was applied to estimate the load history due to the impact of a 9x21 FMJ bullet at 322 m/s. The load history was applied to impact simulations with progressively simplified finite element models. The results demonstrated good adhesion to the results obtained on the same case by means of the already validated FSI method developed by Andreotti et al. [9]. The method allowed to reproduce the dynamic stress condition of the plates both in terms of local stress waves as well as in terms of history of resultant reaction forces. These results demonstrated to be independent from the radius of loaded area in the range of 4.5mm to 6.75mm.

Moreover, the efficiency of the method has demonstrated to be significantly high, with calculation times reduced to less than 0.2% of the time needed by the locally detailed FSI-based method used as a benchmark. The study therefore demonstrates the method as useful for industrial applications and suggests further investigations of its applicability on different ammunitions and targets.

## REFERENCES

- [1] Heimbs, S. (2011). Computational methods for bird strike simulations: A review, *Comput. Struct.*, 89(23–24), pp. 2093–112, Doi: 10.1016/j.compstruc.2011.08.007.
- [2] Parker, S.P. Parker, S.P.(2003). Bullet Splash. Available at: <https://encyclopedia2.thefreedictionary.com/bullet+splash>. [accessed May 29, 2021].
- [3] Quigley, E.F. (1989). EPIC-2 Calculated impact loading history for finite element analysis of ballistic shock, Aberdeen, Maryland.
- [4] Goda, I., Girardot, J. (2021). Numerical modeling and analysis of the ballistic impact response of ceramic/composite targets and the influence of cohesive material parameters, *Int. J. Damage Mech.*, 30(7), pp. 1079–1122, DOI: 10.1177/1056789521992107.
- [5] Goda, I. (2022). Ballistic resistance and energy dissipation of woven-fabric composite targets: Insights on the effects of projectile shape and obliquity angle, *Def. Technol.*, DOI: 10.1016/j.dt.2022.06.008.
- [6] Yunfei, D., Wei, Z., Guanghui, Q., Gang, W., Yonggang, Y., Peng, H. (2014). The ballistic performance of metal plates subjected to impact by blunt-nosed projectiles of different strength, *Mater. Des.*, 54, pp. 1056–1067, DOI: 10.1016/j.matdes.2013.09.023.
- [7] Iqbal, M.A., Diwakar, A., Rajput, A., Gupta, N.K. (2012). Influence of projectile shape and incidence angle on the ballistic limit and failure mechanism of thick steel plates, *Theor. Appl. Fract. Mech.*, 62(1), pp. 40–53, DOI: 10.1016/j.tafmec.2013.01.005.
- [8] Bresciani, L.M., Manes, A., Romano, T.A., Iavarone, P., Giglio, M. (2016). Numerical modelling to reproduce fragmentation of a tungsten heavy alloy projectile impacting a ceramic tile: Adaptive solid mesh to the SPH technique and the cohesive law, *Int. J. Impact Eng.*, 87, pp. 3–13, DOI: 10.1016/j.ijimpeng.2015.10.003.
- [9] Andreotti, R., Abate, S., Casaroli, A., Quercia, M., Fossati, R., Boniardi, M. V. (2021). A simplified ale model for finite element simulation of ballistic impacts with bullet splash – development and experimental validation, *Frat. Ed Integrità Strutt.*, 15(57), pp. 223–245, DOI: 10.3221/IGF-ESIS.57.17.
- [10] LSTC. (2015). LS-DYNA® keyword user’s manual–version R8.0, I.