Visual Abstract

Damage identification in RC bridges by confronting two approaches: visual inspection and numerical analysis

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ABSTRACT. The present article aims to summarize the research study that was conducted the efficiency of methods and techniques designed for the detection and localization of faults in civil engineering structures, particularly in bridge structures. The diagnosis of a real reinforced concrete bridge by a visual inspection is presented. Then, three numerical damage detection and localization methods, namely the eigenfrequency change method, the eigenstrain change method (Coordinate Modal Assurance Criterion – CO-MAC), and the strain energy change method, are explicitly presented. Furthermore, the modeling of the bridge, before and after damage, using the Ansys software was carried out in order to identify all possible bridge defects. Afterward, the numerical results are graphically represented using the above mentioned methods. This made it possible to confirm the initial diagnosis and hence assess the damages observed on site and also in other zones.

KEYWORDS Bridge, Damage, Inspection, Eigenfrequencies, Eigenstrains, Coordinate Modal Assurance Criterion.

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INTRODUCTION

Structures, particularly bridges and viaducts, are essential and indispensable links in basic infrastructure networks. There is no doubt that their durability and functionality represent a major challenge for the public authorities of all countries around the world.

As these structures are high tech constructions that are constantly evolving, they require appropriate advanced technology means that allow performing an accurate diagnosis. It should be noted that appropriate monitoring and implementation of specific inspections can certainly make it possible to schedule some maintenance actions in order to ensure the safety of the bridge for the users and to extend the life of these structures [1].

Researchers and civil engineers have developed several useful techniques that can be employed within the context of structural assessment, and more specifically for the periodic monitoring of structures. Recently, several methods for damage detection in bridges have been developed; some of them are more effective than others [2, 3]. The above

mentioned methods are currently used for the purpose of investigating the behavior of the structure through the change of the modal parameters, such as the eigenfrequencies and eigenstrains [4].
Recently, several works have concerned the continuous monitoring of old or new bridges using the statistical analysis of the recorded vibration signals by optimization algorithms or learning machines to identify any damage [5, 6, 7]. Interesting methods have been developed for monitoring damage in bridges by characterizing damage levels by combining visual inspections and in-situ damages assessment and ambient vibration analysis [8].
This article is structured around two parts. The first one includes an expertise, while a numerical approach is carried out in the second part. In the first stage, a real bridge with reinforced concrete girders, built in 1976, is inspected in detail, in order to establish a diagnosis. Indeed, all the existing apparent degradations at each point of the structure are identified and recorded. Then, during the second stage, three damage detection methods, namely frequency change method, strain change method, and strain energy change method, are applied. In these three methods, it is an essential problem is the modification of the modal parameters of a structure under in order to evaluate the performance level of each of these three methods. Then these methods have been applied using the results from a three-dimentional finite element model, using the Ansys software [9], with the aim of detecting all the degradation that had previously occurred in the structure [10].

DETECTION AND LOCALIZATION OF DAMAGE BY VISUAL INSPECTION

To carry out this experiment, an old road bridge with reinforced concrete beams was selected.

Presentation of the bridge

A reinforced concrete beam bridge was selected for this study. This bridge crosses Oued Aounia (Aounia Wadi) in the Wilaya (province)of Tlemcen (Northwestern Algeria). It is 45 years old; it was built in 1976. It is located at PK (6+750) of the Wilaya Road 101 in the Municipality of Maghnia, as illustrated in Fig. 1. This structure is crossed by an average daily traffic of 5105 vehicles; 4% of them are heavy goods vehicles.

Figure 1: Oued Aounia Bridge.

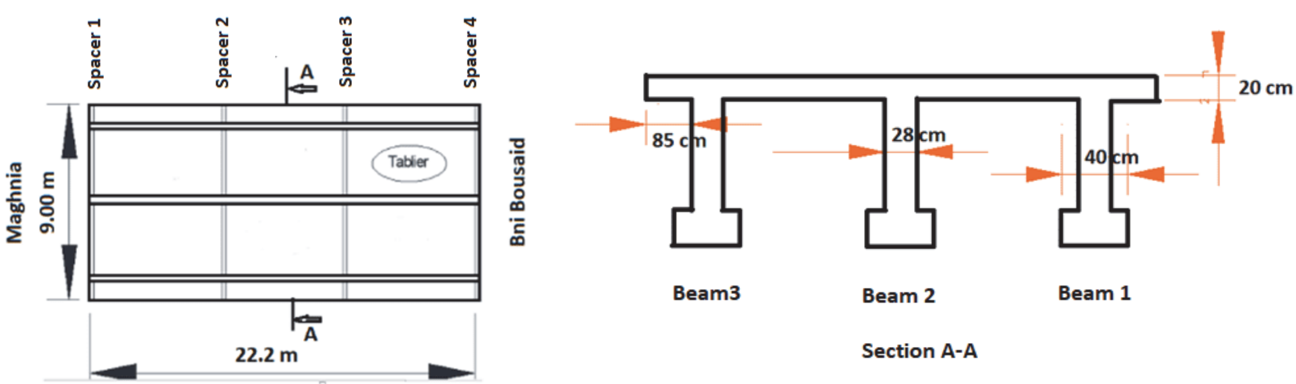
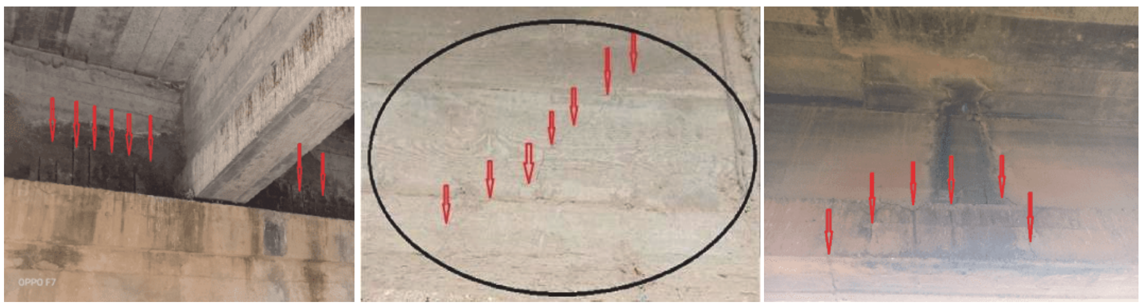


Figure 2: Plan view with a cross section of the bridge superstructure.

The deck of the bridge has a width of 9.00 meters, length of 22.2m, and a thickness of 0.20m. It is supported by three main reinforced concrete beams, 1.35m high, braced at four points, two at the ends and two at intermediate positions; they are separated by distances equal to 7m, 6.4m, and 7m, respectively. The thickness of the slab is 0.20m, as shown in Fig. 2.

Bridge inspection

The expertise carried out on Oued Aounia bridge, allowed us to get the various visible degradations existing on the structural elements of the studied structure. Figs. 3 and 4 show the location of some damaged areas detected during the bridge's inspection.



(a) (b) (c)

Figure 3: Degradation on the lower part of the bridge. (a) Degradation on spacer 4, (b) Degradation on beam 1, (c) Degradation on beam 3.

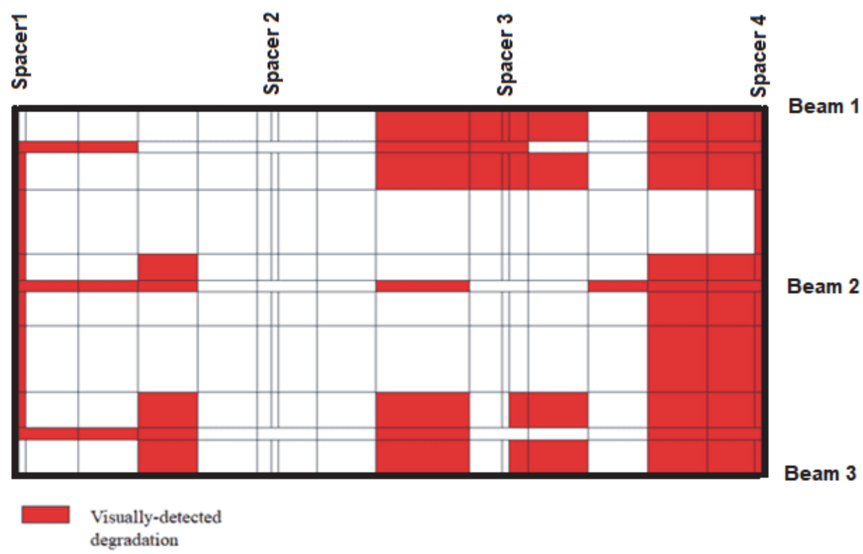


Figure 4: Plan view of the bridge with visual inspection damage identification.

METHODOLOGY ADOPTED FOR THE ANALYSIS OF THE DAMAGED BRIDGE

The procedure adopted for the analysis of damages using an appropriate visual inspection technique, on the one hand, and the automatic detection and localization of defects using the modal characteristics, on the other, are illustrated in the flowchart presented in Fig. 5. This flowchart is employed in the present work. It illustrates the classical modal analysis procedure for a damaged structure using the visually gathered information by an expert engineer. The complementarity between the two approaches makes it first possible to confirm the noted actual damage, second to eliminate the observed superficial damage, and finally to detect any eventual hidden damage or stiffnesses change in the structure, which would have been impossible to identify otherwise.

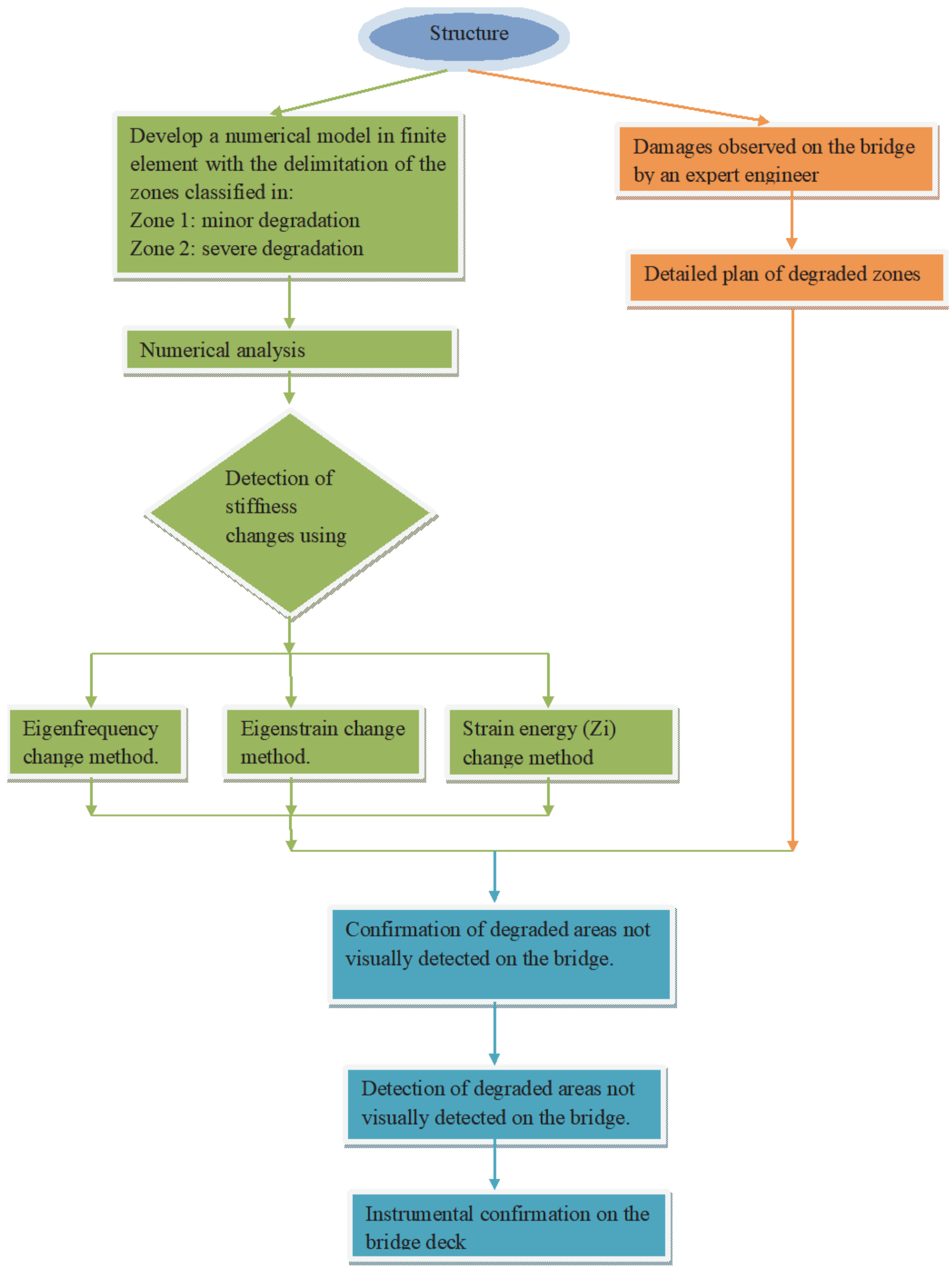


Figure 5: Flowchart for bridge analysis method.

Detection and localization of damage using numerical methods

The eigenfrequency change method

A slight difference between eigenfrequencies is generally interpreted as an indicator of damage. This supposes that any decreasing frequency implies that a defect exists on the structure. This assumption seems logical and obvious because the damage tends to reduce the structural stiffness[11].

The eigenfrequency deviation Δf can be expressed as follows:

$$\Delta f = f_n^s - f_n^d \tag{1}$$

Here f_n^s is the frequency before damage, and f_n^d is the frequency after damage.
Using the deviation between frequencies, one can calculate the frequencies percentage variation Pi (%) by applying the following expression:

$$Pi(\%) = \frac{(f_n^s - f_n^d)}{(f_n^s)}.100 \tag{2}$$

The eigenstrain change method

The information related to eigenstrains is essentially an important data for the localization of the damages in bridges [12]. Starting from this definition, it was decided to use the coordinate modal assurance criterion (CO-MAC) for the detection and localization of disorders in the bridge study.
For the two eigenstrain matrices, ΦA and ΦB, corresponding to two study cases of the same structure, before and after damage, and for the eigenstrain η on each matrix Φ, the CO-MAC coefficient for the element i of the structure may be determined using the following expression:

$$COMACi = \sum_{k=1}^{\eta} \frac{\left| [\Phi A]_k^i . [\Phi B]_k^i \right|^2}{\left([\Phi A]_k^i \right)^2 . \left([\Phi B]_k^i \right)^2} \tag{3}$$

In all cases, the CO-MAC factor can take real values between 0 and 1. First, regarding the limits of this interval, if the CO-MAC coefficient is equal to 1, one can say that the correlation between the two series of calculations, before and after damage, is complete. However, if this coefficient is equal to zero, this correlation does not exist. Regarding the intermediate values, between 0 and 1, one can easily notice that the correlation is incomplete since the presence of a structural anomaly can modify the modal strains [13].

The strain energy change method

The strain energy variation approach can be used as a means for detecting and locating disorders in civil engineering structures. The principle of this technique consists in seeking the element that has a significant amount of strain energy [14]. This method is mainly applied to one-dimensional finite elements, such as beams, but can also be applied to three-dimensional structures[15].
The combination of modal strains with the bending stiffness of a beam of length l gives the strain energy using the following expression[12]:

$$U = \frac{1}{2}\int_0^l EI(x)\left(\frac{\delta^2(v)}{\delta x^2}\right)^2 .dx \tag{4}$$

Furthermore, the strain energy of the beam and that j-th element of the beam, for the same vibration mode, before the damage is:

$$U_i = \frac{1}{2}\int_0^l EI(x)\left(\frac{\delta^2 \Phi_i(\mathrm{x})}{\delta x^2}\right)^2 .dx \tag{5}$$

$$U_{ij} = \frac{1}{2}\int_0^{lj} EI_j(x)\left(\frac{\delta^2 \Phi_i(\mathrm{x})}{\delta x^2}\right)^2 .dx \tag{6}$$

In the same way, the strain energy of the beam and that of the j-th element of the beam after the damage are respectively given by:

$$\dot{U}_i = \frac{1}{2}\int_0^l \dot{E}I(x)\left(\frac{\delta^2 \dot{\Phi}_i(x)}{\delta x^2}\right)^2.dx \tag{7}$$

where δ is the partial derivative

$$\dot{U}_{ij} = \frac{1}{2}\int_0^{lj} \dot{E}I_j(x)\left(\frac{\delta^2 \dot{\Phi}_i(x)}{\delta x^2}\right)^2.dx \tag{8}$$

After the manipulation of the eigenstrains and strain energies, the damage indicator for a j-th element is written as follows:

$$\beta_{j=}\frac{\dot{U}_{ij}/\dot{U}_i}{U_{ij}/U_i} \tag{9}$$

Now, consider the damage indicator βj, the normalized and generalized parameter of defects in the structure can then be written as:

$$z_j = \frac{\beta_j - \overline{\beta}}{\sigma_\beta} \tag{10}$$

Here $\overline{\beta}$ and σ_β are, respectively, the mean value and the standard deviation of βj.
Any positive value of the indicator Zj indicates damage.

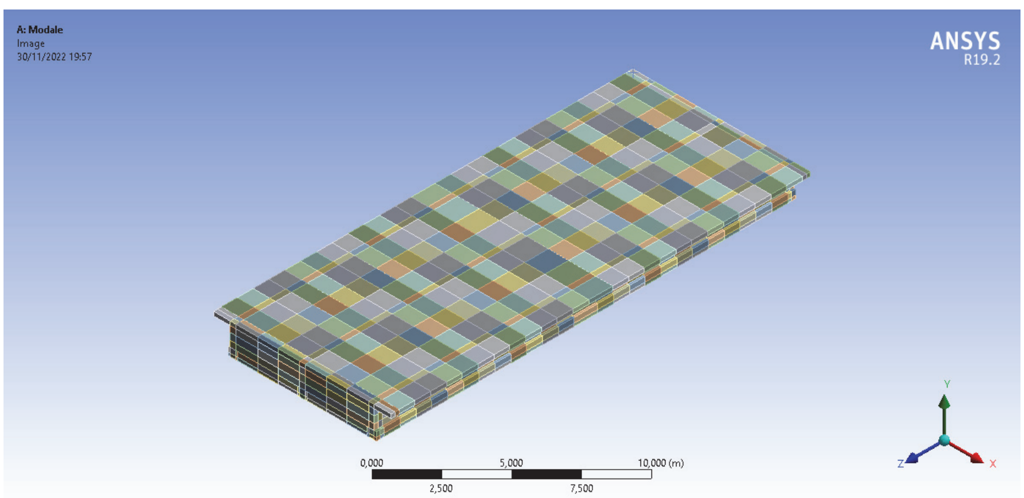


Figure 6:3D numerical model of the bridge.

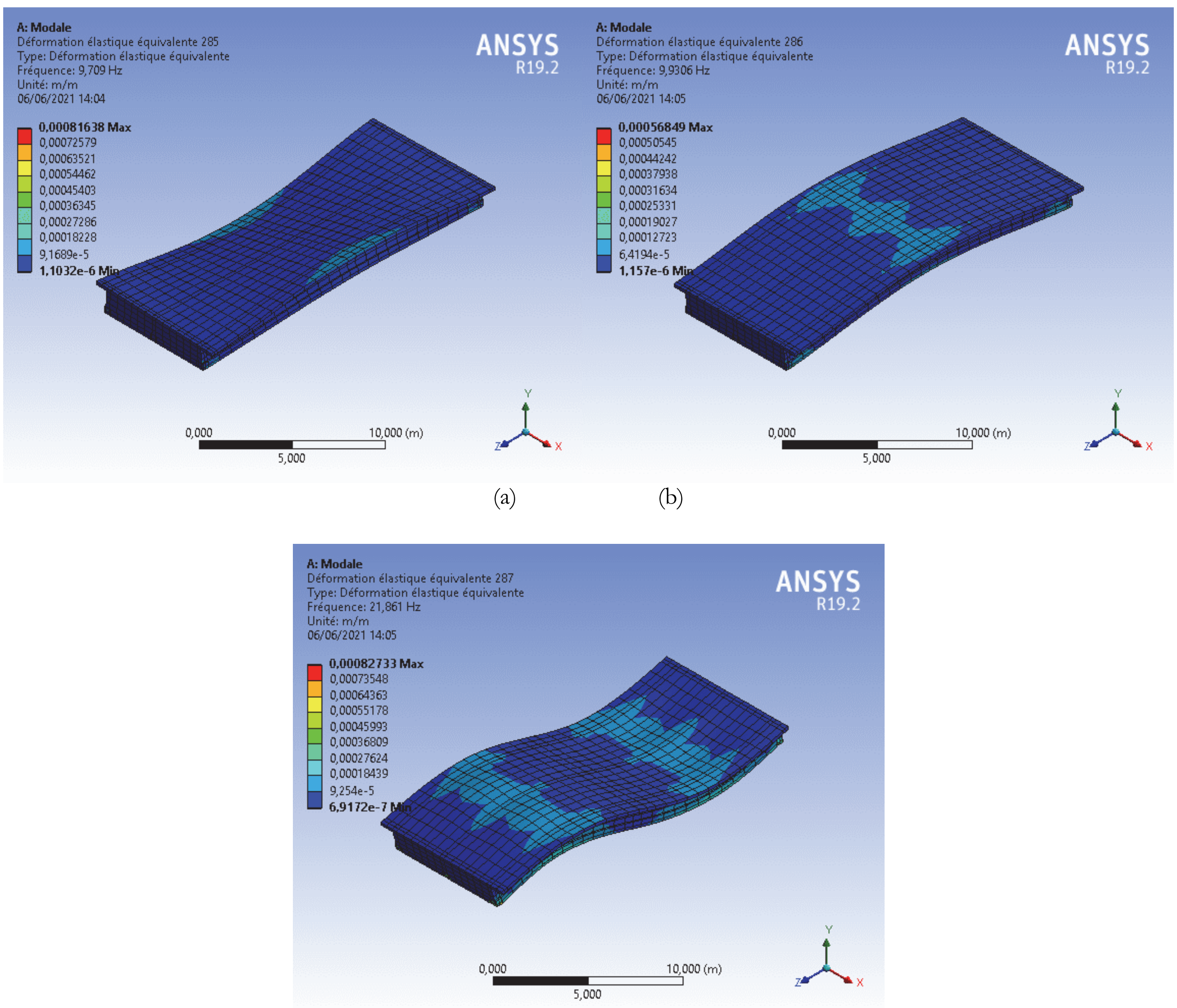
NUMERICAL MODELING OF THE BRIDGE

Most of the methods intended for the detection and localization of damages in a structure are based on the change in its modal parameters (eigenfrequencies, eigenstrains, etc.). These parameters can be deduced from the finite element numerical model. The physical and mechanical characteristics of the used material as concrete are: Density=2.50; Young modulus E=32000 MPa and Poisson coefficient ν=0.30.

A three-dimensional numerical model of the bridge was established using 263 three-dimensional finite elements with 8 nodes and 3 DOF per node and a total of 6312 DOF. Concerning the supports conditions of the bridge, the translation DOFs are fixed and the rotational DOFs are free.
The results of the modal parameters extracted from the dynamic model are stored in Excel files. For each finite element of the bridge, a column matrix of the eigen mode of vibration has been recorded, i.e., 263 column matrix is stored. This operation was done for the first 100 vibration modes. A total of 26300 column matrix of eigen modes of vibration is tidy in text files using a python program[16]. This same procedure was applied to the bridge in both states: the structure in the healthy state and in the damaged state, in order to process the results a second time with the Python program to extract the coefficients indicating damage. Fig. 6 shows the 3D numerical model discretized into 263 elements:

Results of the vibration eigenmodes of the model before damage

As it is not possible to represent the 100 vibration eigenmodes, it was decided to select the most significant eigenmodes of vibration that represent the contribution of the most important masses, as it is clearly illustrated in Figs. 7 and 8.



(a) (b)

(c)

Figure 7:Modal strains of the structure before damage.(a)Mode 1: f = 9.70Hz, (b) Mode 2: f = 9.93Hz (c), Mode 3: f = 21.86Hz

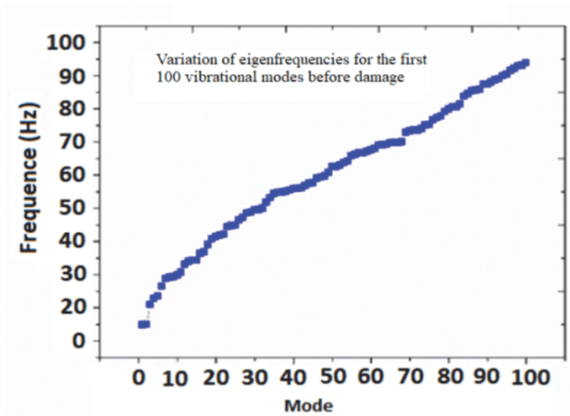


Figure 8:Variation of eigenfrequencies for the first 100 vibrational modes before damage.

Results of the vibration modes of the model after damage

The following Figs. 9 and 10 show the first three modes of vibration with their eigenfrequencies; these figures display the participations of the highest masses.

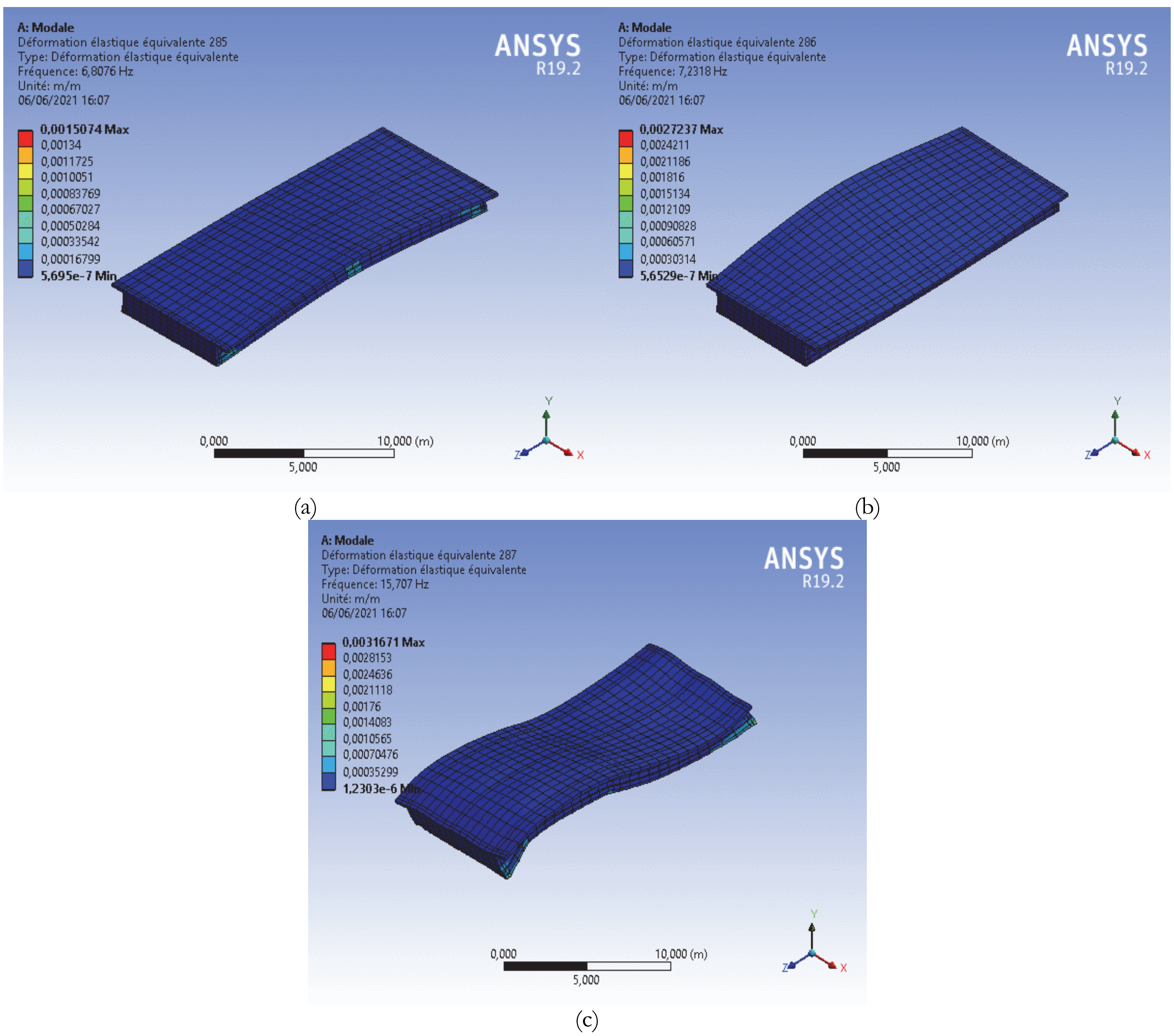


Figure 9: Modal strains of the structure after damage.(a)Mode 1: f = 6.80 Hz, (b) Mode 2: f = 7.23 Hz, (c) Mode 3: f = 16.01 Hz.

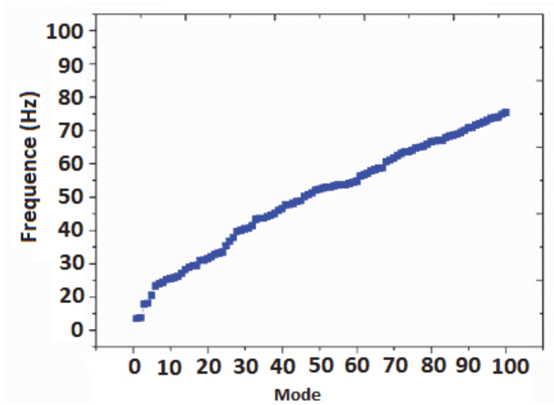


Figure 10:Variation of eigenfrequencies for the first 100 vibrational modes after damage.

APPLICATION OF METHODS FOR BRIDGE DAMAGE ASSESSMENT

The previously determined results by visual inspection and numerical analysis of the bridge can help, using the above mentioned damage assessment methods, to detect and locate the existing disorders on the structure [2].
Through several works carried out in the field of auscultation of the structures, the 'serious damages' are the disorders located in critical places of the structure with a crack thickness greater than 0.5 mm, for example those on the beams near supports, in addition to the cracks in the beams inclined at 45° and the cracks observed in the tension zones of the beams [17]. For the disorders judged as 'weak damage', they are the superficial cracks with crack's thickness lower than 0.5 mm in particular those observed in the slab of the deck and some on the compressed parts of the beams.
In the following section, the three damage detection methods, namely the frequency change method, the CO-MAC method, and damage indicator method, are used to assess and locate the damage existing on the structure.
In order to avoid overlaps on the results of the histograms of the CO-MAC coefficient and that of the damage indicator, the structure is discretized into 4 spacers of 30 finite elements each, 3 beams of 26 finite elements each, and the upper slab with 65 finite elements.
The Tab. 1 shows the two classes of damage on the elements of the bridge inspected (minor damage and severe damage).

Rate	Definition
1	Minor damage
2	Severe damage

Table 1:Classification of damage level.

Tabs. 2, 3, 4 and 5 show the damage levels of the impaired areas on each element of the bridge.

• Beams

	Beam 1													
Damaged elements	1	2	3	4	13	14	15	16	21	22	23	24	25	26
Damage level	2	2	2	2	2	2	1	1	1	1	2	2	2	2

Table 2: Number of damaged elements in the beam1 with their damage levels.

	Beam 2													
Damagedelements	1	2	3	4	5	6	13	14	21	22	23	24	25	26
Damage level	2	2	2	2	2	2	2	2	2	2	2	2	2	2

Table 3: Number of damaged elements in the beam2 with their damage levels.

	Beam 3													
Damaged elements	1	2	3	4	5	6	13	14	21	22	23	24	25	26
Damage level	2	2	2	2	2	2	2	2	2	2	2	2	2	2

Table 4: Number of damaged elements in the beam3 with their damage levels.

• Spacer

It was visually noticed that the two extreme spacers of the structure, spacers 1 and 4, are completely damaged (bursting of concrete and appearance of main reinforcement). For the two intermediate spacers, no damage was visually detected.

• Deck

The damage could only be observed around the periphery of the deck. It should be noted that the deck is covered with the asphalt concrete wearing course.

Elements	Deck																
Damaged elements	20	31	35	36	45	46	50	56	57	58	59	60	61	62	63	64	65
Damage level	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Table 5: Numbers of damaged elements in the deck with their damage levels.

Detection and localization of damage using numerical methods

The eigenfrequency change method

Fig. 11 gives a comparison of eigenfrequencies before damage and after damage of the bridge.

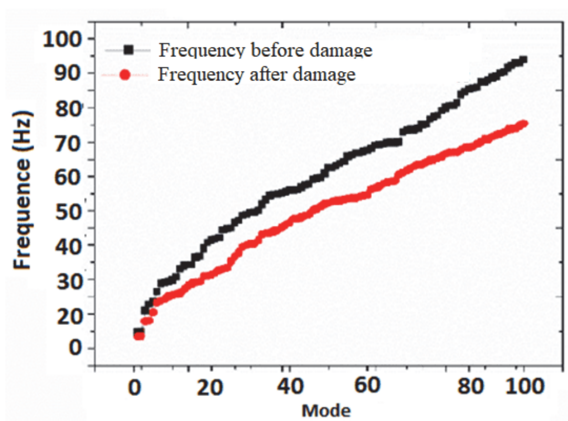


Figure 11: Deviation of eigenfrequencies for the first 100 vibration eigenmodes of the bridge, before and after damage.

The modal analysis of the girder bridge for the first 100 modes, before and after damage, reveals that the eigenfrequency change method is quite effective in detecting damages. The difference of eigen periods is 2.9 sec (f1 = 0.344 Hz) for the first mode, whereas it is 37 s for the 100th eigenmode (f100 = 0.027 Hz). This shows that the detection level of this method is proportional to the number of vibration eigenmodes.

The eigenstrain change method

The histograms of the CO-MAC coefficient calculated numerically by a code developed in Python [16] are presented in the following section. The calculation of the CO-MAC coefficient was carried out for the first 100 eigenmodes of vibration. For each element, only the histogram with the highest level of damage detection is presented (Figs. 12 to 19).

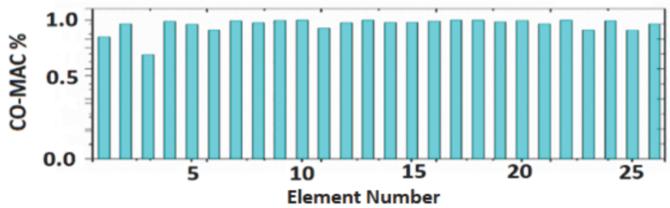


Figure 12: Histograms of the CO-MAC coefficient for Beam 1 and Mode 73.

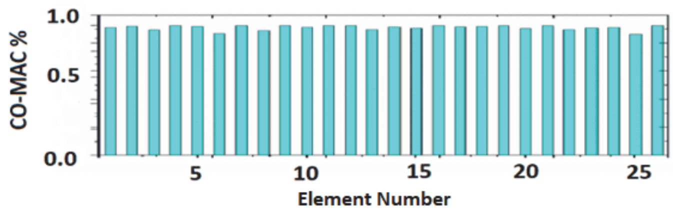


Figure 13: Histograms of the CO-MAC coefficient for Beam 2 and Mode 59.

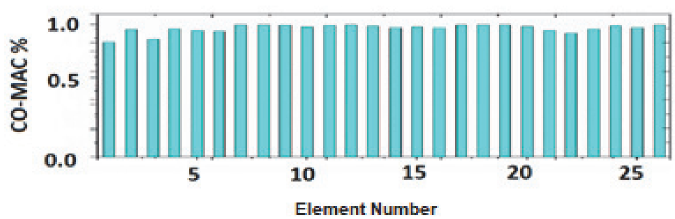


Figure 14: Histogram of the CO-MAC coefficient for Beam 3 and Mode 59.

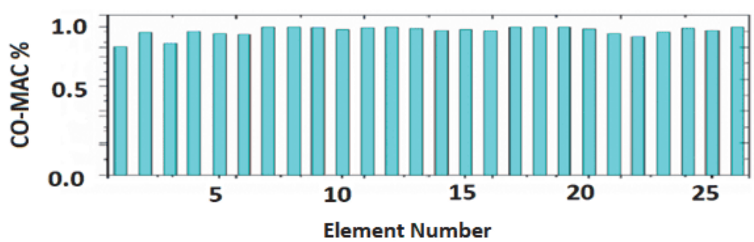


Figure 15: Histogram of the CO-MAC coefficient for Spacer 1 and Mode 98.

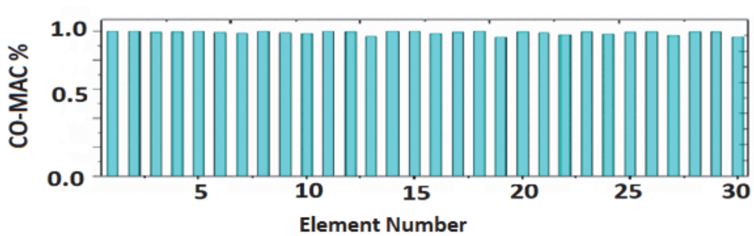


Figure 16: Histogram of the CO-MAC coefficient for Spacer 2 and Mode 100.

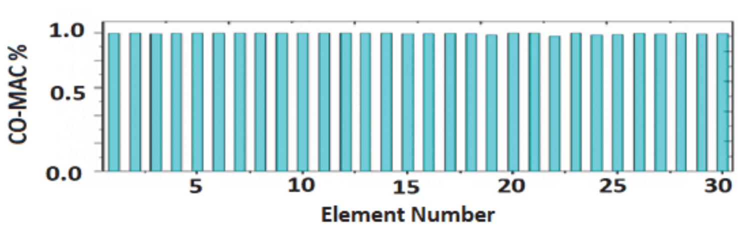


Figure 17: Histogram of the CO-MAC coefficient for Spacer 3 and Mode 100.

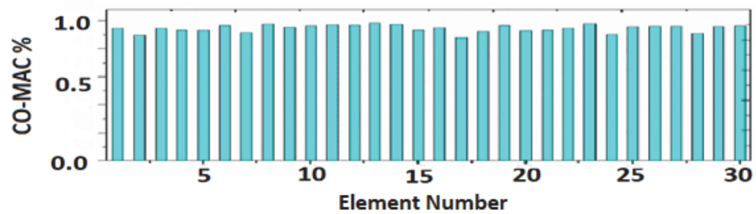


Figure 18: Histogram of the CO-MAC coefficient for Spacer 4 and Mode 98.

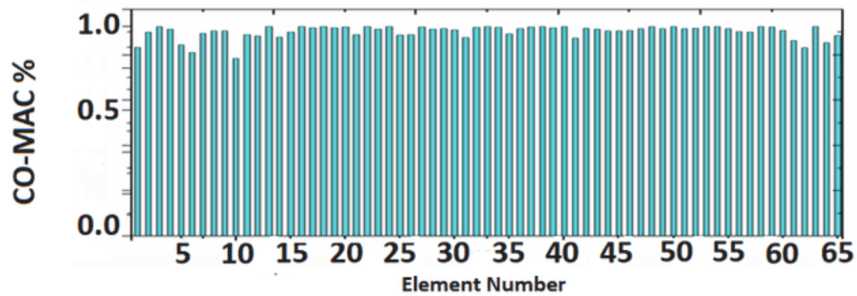


Figure 19: Histogram of the CO-MAC coefficient for the deck and Mode 100.

The detection and localization of damages on the structure is always carried out by the eigenstrain change method (CO-MAC). The obtained results showed that in order to achieve a high level of damage detection on all the elements of the bridge, one needs to have a large number of vibration modes.
For example, it was necessary to apply 100 natural modes of vibration in order to achieve a detection level of 100 % on the spacer elements, 70% on the beam elements, and 94% on the deck.
<u>Strain energy change method</u>
It is worth indicating that the discretization of the elements constituting the bridge made it possible to draw the histograms of the coefficients of the indicator Zi for which the damage level is high quite. They are illustrated in Figs. 20 to 27.

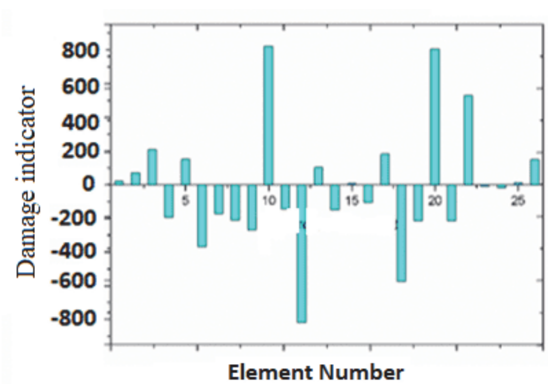


Figure 20: Histogram of the damage indicator for Beam 1 and Modes 1 to 5.

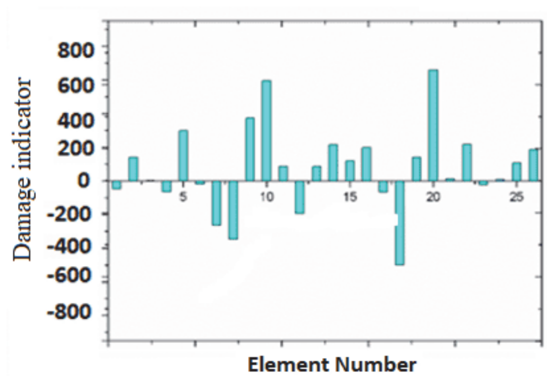


Figure 21: Histogram of the damage indicator for Beam 2 and Modes 1 to 5.

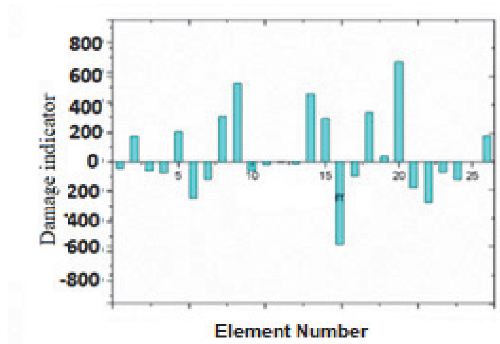


Figure 22: Histogram of the damage indicator for Beam 3 and Modes 1 to 5.

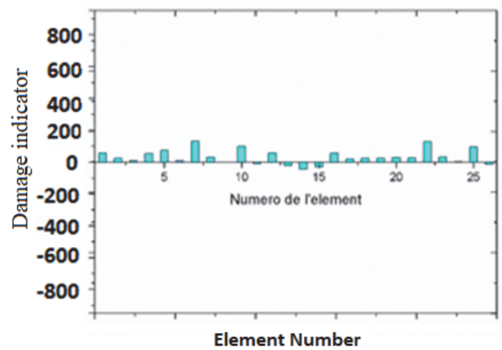


Figure 23: Histogram of the damage indicator for Spacer 1 and Modes 1 to 5.

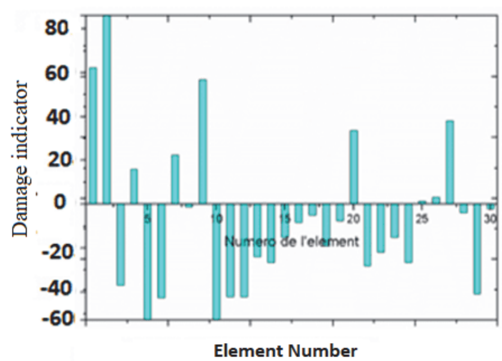


Figure 24: Histogram of the damage indicator for Spacer 2 and Modes 1 to 5.

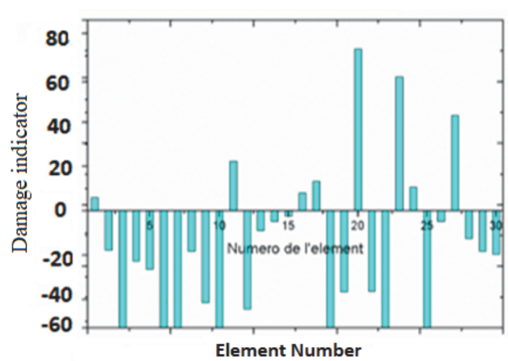


Figure 25: Histogram of the damage indicator for Spacer 3 and Modes 1 to 5.

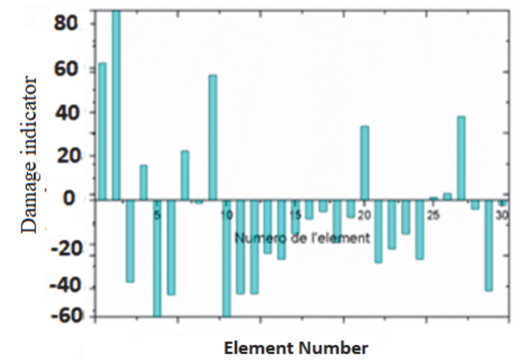


Figure 26: Histogram of the damage indicator for Spacer 4 and Modes 1 to 5.

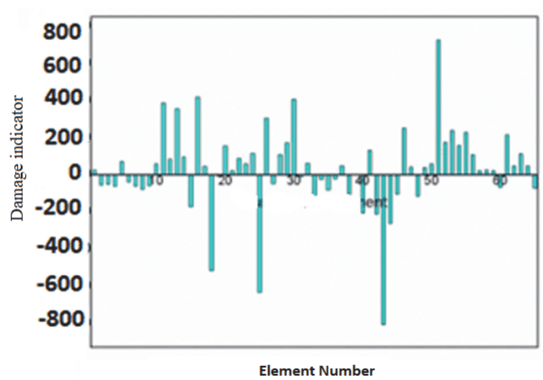


Figure 27: Histogram of the damage indicator for the deck and Modes 1 to 5.

The results obtained show that the strain energy change method is highly effective for the detection and localization of damages. The advantage of the method lies in the fact that only a limited number of vibration modes can achieve a high level of fault detection and localization. For example, in the present case, detection levels of 70%, 64%, and 74% were achieved in the spacer elements, beam elements, and deck, respectively, beyond the fifth eigenmode of vibration. Fig. 28 shows the location of degraded areas numerically recorded.

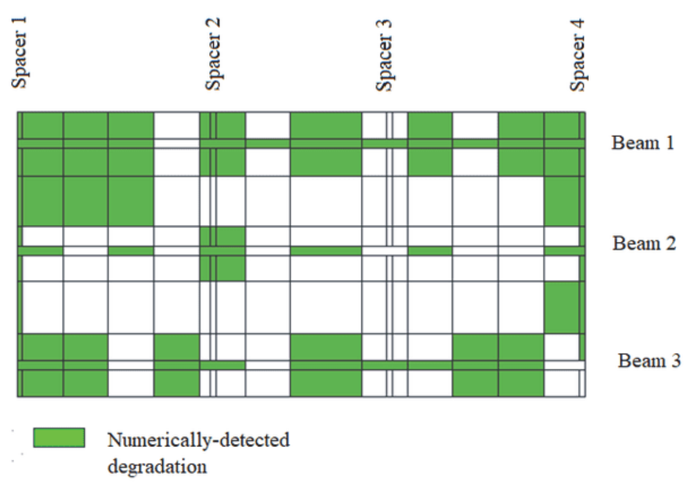


Figure 28: Plan view of the bridge with details of numerically recorded damage.

Comparison of results for the detection of stiffness change in the structure using the visual approach and numerical approach

The previously obtained results suggest that the structure stiffness depends on the approach adopted, i.e. visual approach and numerical approach. Three (03) scenarios can therefore be envisaged for the detection of anomalies.

- First scenario

The damage is detected through visual inspection but not through digital inspection (element 4 in beam 1). This implies that the disorder is only superficial.

- Second scenario

The damage is not detected through visual inspection but is detected through digital inspection (element 6 in beam 1). This reveals the presence of invisible to the naked eye damage.

- Third scenario

The damage is detected both through visual inspection and digital inspection, which means that the observed anomaly visually is digitally confirmed as well. Fig. 29 shows the localization of the damage recorded by the two visual and numerical approaches.

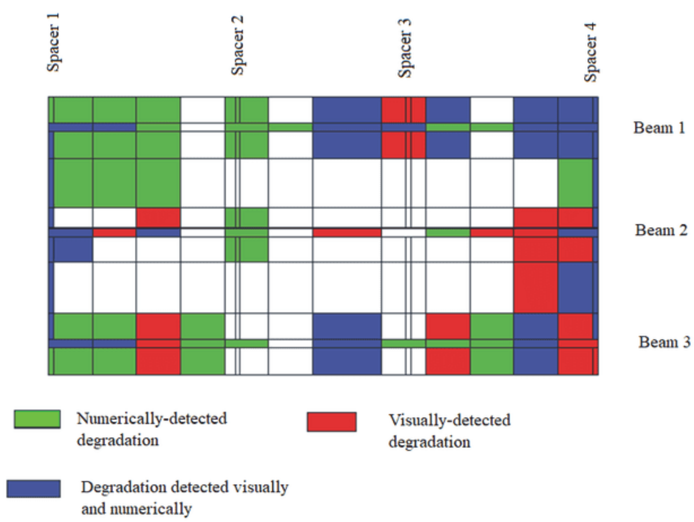


Figure 29: Plan view of the bridge with details of the damage recorded by the two visual and numerical approaches.

Conclusion

The present article aims primarily to design a high-performance method for detecting and locating damage in civil engineering structures based on the visual observation of an expert engineer and also on the numerical analyses that are commonly performed.

The expertise component depends first on the engineer's ability to observe the structures to be examined, and then on his mastery of the design and behavior of the bridge structure.

However, the digital component depends on the good mastery of calculation methods, such as the frequency change method, the strain change method, and CO-MAC method, and also on the detailed analysis of the results obtained from the modeling of structures.

The adopted research approach was applied to an old reinforced concrete girder bridge. The visual inspection operation was carried out on that bridge and all the visible damages were recorded.

It should be emphasized that the 3D finite element model was used to extract the dynamic characteristics of the bridge over the first 100 vibrational modes (eigenfrequencies and eigenstrains). Note also that, in general, the required number of modes depends on the size and complexity of the inspected structure. Subsequently, the three numerical methods for frequency analysis were used to identify the damage percentages of each finite element of the bridge.

At the end of this analysis, and after comparison with the visual damage identification results, it can be concluded that there is a good correlation between the detection of damages by visual inspection and the one by numerical analysis. Therefore, this numerical damage analysis can be applied to confirm the visual diagnosis made by an expert engineer. Thus, the degradations detected visually are in fact real damages (stiffness reduction). This approach can help to detect other surface degradations that have no major impact on the stiffness of the bridge; it can also be used to numerically discover new damages that are not visible to the naked eye. Therefore, it is up to the engineer to quantitatively and qualitatively assess the reliability of the obtained results.

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