On the relationship between J-integral and CTOD for CT and SENB specimens

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ABSTRACT. In this investigation the relationship between *J*-integral and CTOD is studied considering a Compact Tensile (CT) and Single edge notched bend (SENB) specimens using finite element analysis. The magnitude of CTOD is estimated by 90°-intercept method and also by plastic hinge model. The results indicate that there exists a discrepancy in estimation of CTOD by 90°-intercept method and by plastic hinge model. The CTOD values obtained by both the methods are found to be linearly proportional to *J*-integral. The linear proportionality constant d_n between CTOD and J is found to be strongly depend on the method of estimation of CTOD, specimen geometry and *a/W* ratio of the specimens.

KEYWORDS: Crack-tip opening displacement, Finite element analysis, *J*-integral, Plastic hinge model

NOMENCLATURE

a	crack length
b	un-cracked ligament
B	thickness of the specimen
CMOD	crack mouth opening displacement
d_n	constant in relation between J and δ
E	Elastic modulus of the material
J	J- integral parameter
m	constant in relation between J and δ
N	Ramberg-Osgood (R-O) material strain hardening parameter, referred as R-O constant
r	rotation factor
W	width of the specimen
δ , CTOD	crack-tip opening displacement
δ_c	critical crack-tip opening displacement
$(\delta)_{PH}$	crack-tip opening displacement estimated by plastic hinge model
$(\delta)_{90°}$	crack-tip opening displacement estimated by 90° intercept method
σ_y	yield stress of the material
ν	Poisson's ratio

1. INTRODUCTION

Elastic plastic fracture mechanics (EPFM) is the domain of fracture analysis, which considers extensive plastic deformation ahead of crack-tips prior to fracture. It is well known that *J*-integral (*J*) and crack-tip opening displacement, CTOD (δ) can be used as fracture parameters for analysis of fracture problems under EPFM. In EPFM it is required that J and δ should be interchangeable to each other. Thus, it is essential to examine the relation between J and δ. A well-known general relation between J and δ [1] is:

$$J = m\sigma_y\delta \tag{1}$$

where, σ_y is the yield stress of the material, *m*- constant. Earlier references [2-4] indicate that the load intensity measured in terms of *J*-integral as a single parameter alone does not describe the stress/strain field ahead of the crack-tip uniquely and accurately. Hence, there is a necessity of introducing a second parameter with J, which is required to characterize the crack-tip fields. This discrepancy in characterizing the crack-tip fields is due to varied constraint effects in fracture. The factor m in relation between J and δ given in Eq. 1 is known to be constraint dependent [4]. Thus m can serve as a parameter to characterize constraints [4]. Therefore the study of constant m in relationship between J and δ is important in EPFM analysis.

Shih [5] has shown that the relationship between J and δ can be obtained theoretically by Hutchinson-Rice-Rosengren (HRR) stress field equations [6, 7] as:

$$\delta = d_n \frac{J}{\sigma_y} \tag{2}$$

where d_n is a constant which depends on Ramberg-Osgood (R-O) constant N of the material. From Eq.1 and Eq.2 the relation between m and d_n is:

$$d_n = \frac{1}{m} \tag{3}$$

Shih [5] has also shown that d_n usually varies between

0.4 to 0.8 for common structural steels and for elastic-perfectly plastic materials ($N = \infty$) d_n=1, which is obtained by extrapolation. As m or d_n can be used as a constraint parameter [4], it is required to examine the effect of specimen geometry and a/W ratio on the factor d_n, which can address in-plane constraint effects. Panontin et al. [8] have studied the effect of specimen a/W on relationship between J and CTOD under 2D plane strain conditions. But the results of Panontin et al. [8] cannot be useful in fracture analysis of thin sheets. Recently, Kulkarni et al. [9] and Kulkarni et al. [10] have shown that fracture analysis of thin sheets can be done using critical CTOD, δ_c. In their analysis, critical CTOD is computed by the relation between J and δ suggested by Shih [5]. The thin sheet analysis is considered as fully plastic case and d_n in Eq. 2 is taken as 1. As the factor d_n is dependent on Ramberg-Osgood (R-O) constant, N, of the material [5, 8] we feel it is not appropriate to assume the value of d_n=1 for the analysis of thin sheets. Using the value of d_n=1 may lead to some errors in relation between J and δ. This study demands detailed analysis of relationship between J and δ in various specimens under plane stress condition. In fracture analysis, the magnitude of δ can be estimated by plastic hinge model [11] and by 90° intercept method [5]. Hence, the consistency in measurement of CTOD by both the methods is to be studied. The objective of the present investigation is to examine the relationship between the J and δ for CT and SENB specimen and to study the effect of specimen geometry and a/W ratio on magnitude of d_n computed by plastic hinge model and 90° intercept method.

2. FINITE ELEMENT ANALYSIS

The general-purpose finite element code ANSYS [12] is used in this study. Compact Tension (CT) and single edge notch bend (SENB) fracture specimen geometries have been considered in the present investigation. The dimensions of CT and SENB specimens have been computed according to ASTM standard E1290-99 [11] with width of the specimen W=20mm and thickness B=3mm. To study the effect of crack-length on

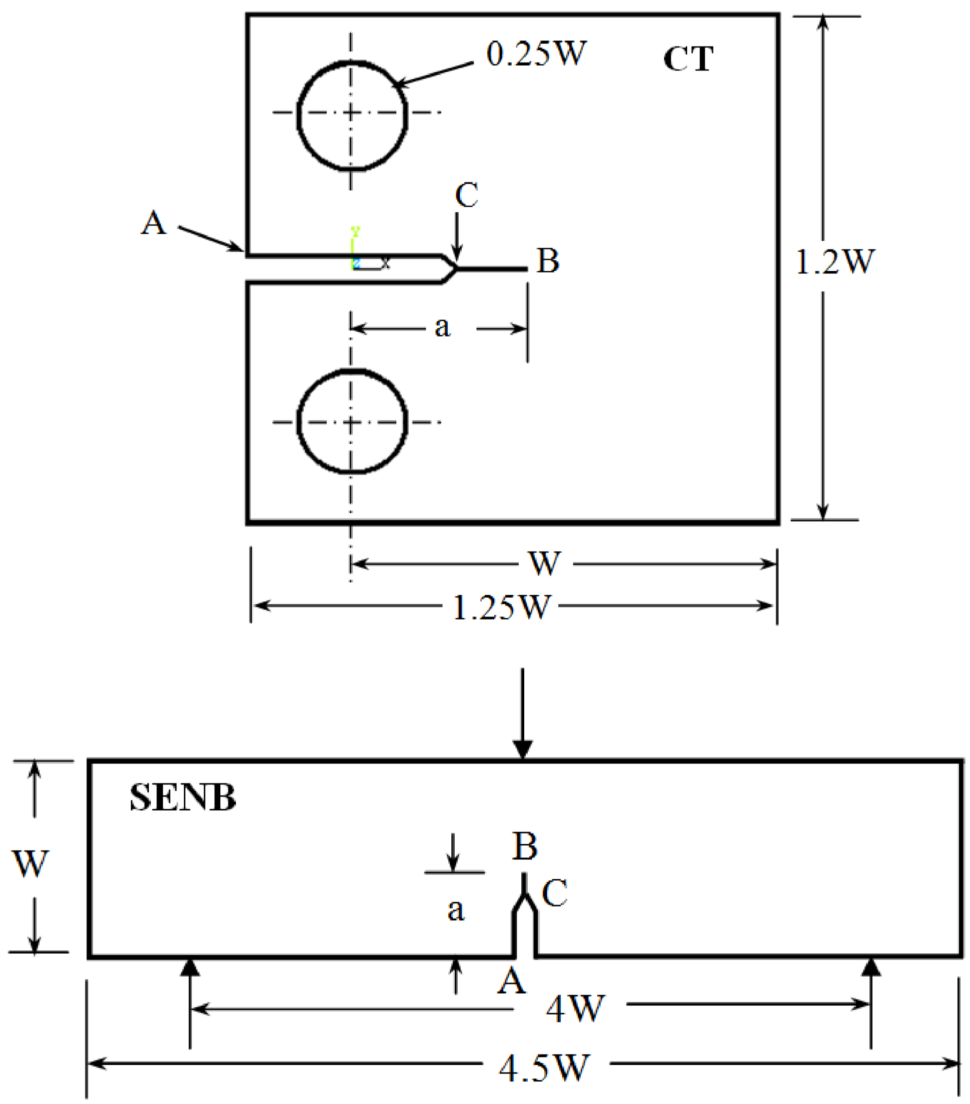

Figure 1: Configurations of CT and SENB specimens used in the analysis.

relationship between J and CTOD, several specimens with crack length to width ratio, a/W= 0.40 to 0.60 in steps of 0.05 have been considered for finite element analysis (FEA). Typical specimen configurations used in this analysis are shown in Fig.1. Only one half of the specimens have been considered for FEA due to the geometrical symmetry. The analysis domain is discritized using eight noded isoparametric quadrilateral elements. The number of elements used in the FEA was 1184 and 1111 for CT and SENB specimens respectively. Typical finite element meshes generated for CT and SENB specimens are shown in Fig.2. In these finite element calculations, the material behaviour has been considered to be multilinear kinematic hardening type pertaining to an interstitial free (IF) steel possessing yield strength (σ_y) of 155 MPa, Elastic modulus (E) of 197 GPa, Poisson's ratio ν = 0.3, and Ramberg-Osgood constants, N=*3*.358. The material data of IF steel used in this analysis has been taken from the earlier report of Kudari et al. [13]. A series of elastic-plastic stress analyses on CT and SENB specimens (Fig.1) of thickness 3 mm and a/W=*0*.40 to 0.60 in steps of 0.05 are carried out for different applied load levels. In these analyses, for every load steps, elastic-plastic fracture parameters J-integral and CTOD by 90° intercept method and plastic hinge model have been computed.

J-integral

The magnitude of J-integral has been evaluated for a path at a specific loading condition using the expression suggested by Rice [14]:

$$J = \int_{\Gamma} \left(W dy - T_i \frac{\partial u_i}{\partial x} ds \right) \tag{4}$$

$$W = \int_0^{\varepsilon} \sigma_{ij} d\varepsilon_{ij} \qquad T_i = \sigma_{ij} n_j$$

where, W=strain energy density, T_i = traction vector, u_i=displacement vector, s = element of arc length along contour . An anticlockwise contour around the crack-tip in an analysis domain of a specimen (Refer Fig.2 for a typical contour) is considered for estimating the intensity of J-integral. In this analysis, it was noted that there exists some deviation between the magnitudes of J estimated using different paths. Hence, magnitude of J for a particular loading condition in a specimen has been calculated by considering four different contour

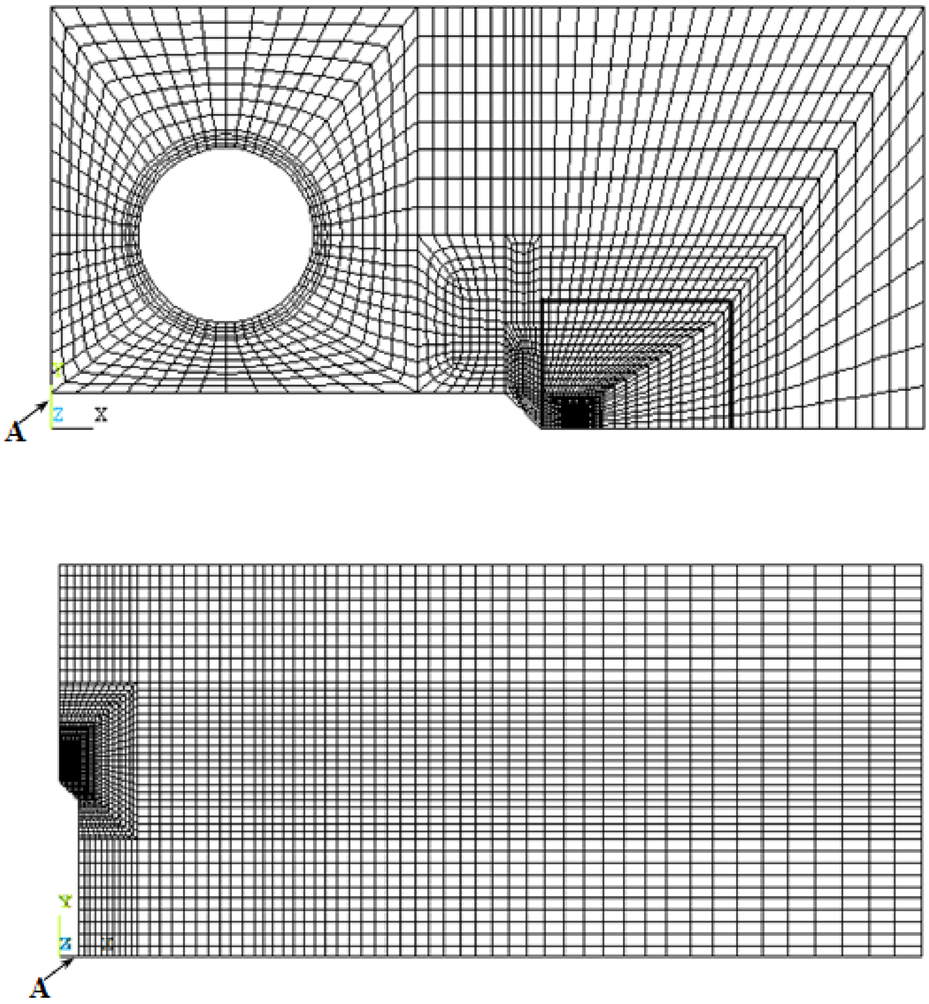

Figure 2: A typical FE mesh used for the analysis of CT and SENB specimens.

paths. The mean value of computed J has been considered for the analysis. The values of the coefficient of variation (CEV) (equal to the standard deviation divided by the mean) associated with the estimated mean values of J were computed, the details of which is discussed elsewhere [13].

Crack-tip opening displacement (CTOD)

The magnitudes of CTOD for various load steps have been estimated by two methods: (i) by 90°-intercept method [5] and (ii) by conversion of crack mouth opening displacement (CMOD) to CTOD using rotation factor, which is popularly referred as plastic hinge model and used in experimental fracture analysis [11]. In 90° intercept method, for every load step, the y-displacement of each node on the crack flank BC for CT specimen and x-displacement of each node on the crack flank BC for SENB specimen (Fig.1) is plotted. According to this method an intercept of a 45° line drawn from the crack-tip with the crack flank displacement plot is considered as half part of CTOD. In the plastic hinge model CMOD is converted to CTOD using rotation factor. At each applied load the magnitude of half CMOD is noted from the y-displacement of the node at point A for CT and the x-displacement of the node at point A for SENB specimen (Fig.2). The CMOD data obtained from FE results is then used to compute the magnitude of CTOD using a relation given in ASTM E1290-99 [11]:

$$\delta = \frac{(CMOD).r.b}{a + r.b} \qquad (5)$$

where r is rotation factor, the value of r according ASTM E1290-99 [11] varies with specimen a/W ratio and is between 0.44-0.47 and 0.44 for CT and SENB specimens respectively, b is the ligament and a is the crack length of the specimen. The Eq. 5 estimates only the plastic part of CTOD, as the investigation is on thin sheets (plane stress elastic-plastic analysis); the elastic part of CTOD is found to be insignificant and is neglected in the present work.

3. RESULTS AND DISCUSSION

Various load steps have been applied on the specimen with a/W = 0.5 to study the stress distribution in the specimen analysis domain. At each load step the magnitude of J-integral has been computed using Eq. 4. As discussed in section 2 the value of CMOD/2 is noted from the displacement of the point A as shown in Fig.2. The CMOD values estimated in CT and SENB specimens by FEA are plotted against J in Fig.3. This figure shows variation of CMOD with respect to the loading parameter J for both the specimen is linear. This figure also indicates that the magnitude of CMOD in CT specimen is more than that of SENB specimen for similar value of J. The difference in CMOD between both the specimens is found to increase as loading parameter J-increases. The results plotted in Fig.3 apparently shows that the relationship between J and CTOD is specimen geometry dependent. The CMOD data estimated by FEA is used to compute the magnitude of CTOD using a Eq. 5. The magnitude of CTOD is also computed by 90°-intercept method [5]. In this method, for every load step, the displacement of

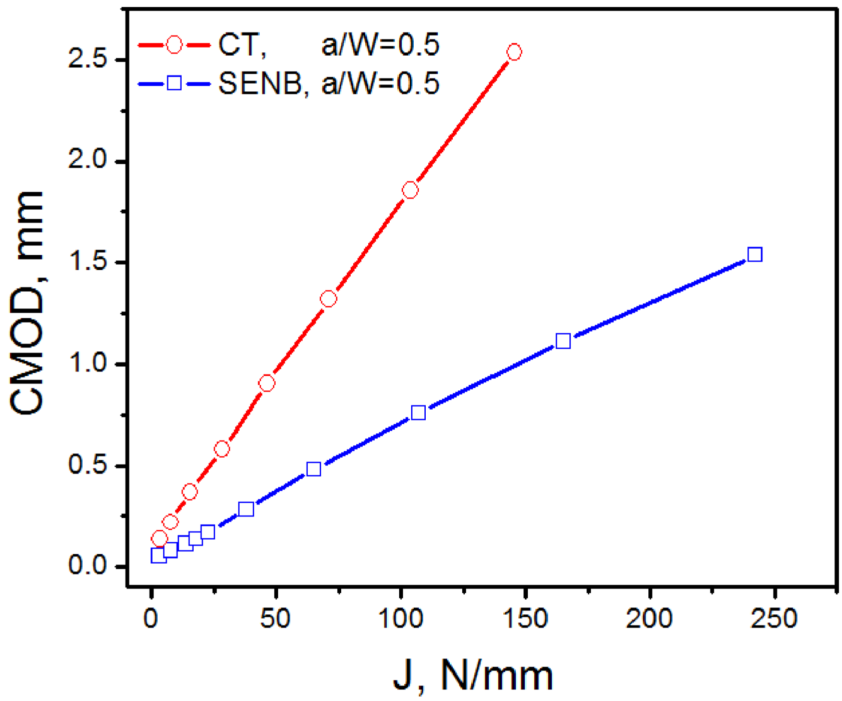

Figure 3: Variation of J *vs.* CMOD for CT and SENB specimen.

each node on the crack flank BC (Fig.1) is listed with the help of ANSYS post processor and plotted using grapher software. Such a typical plot for CT specimen for various values of J is shown in Fig.4. The intercept of y-displacement of crack flank with the 45° lines drawn from the crack-tip is taken as half part of CTOD as indicated in Fig.4.

The magnitudes of CTOD calculated from both the methods have been plotted against J/σ_y in Fig.5 and Fig.6 respectively for CT and SENB specimens. It is interesting to know from these figures that the variation of δ against J/σ_y is linear; this nature of variation of δ against J/σ_y is in good agreement with the results of Panontine et al. [8]. It is also clear from Fig.5 and Fig.6 that the magnitudes of δ obtained from plastic hinge model and 90° intercept method differ with respect to J/σ_y. This discrepancy may be attributed to the methods of computing the CTOD (δ). In reality it is difficult to estimate the exact value of δ. The 90°-intercept method is a theoretical method of estimating δ by constructing a 90° triangle at the crack-tip. This method is difficult to use in an experimental fracture analysis. An attempt of using this method of estimating δ is carried out by Kulkarni et al. [9]. The second method based on plastic hinge model is popularly used in experimental methods of fracture analysis. In this method a clip gauge is used to measure CMOD. The obtained CMOD is then converted to CTOD, δ, by considering the deflection of

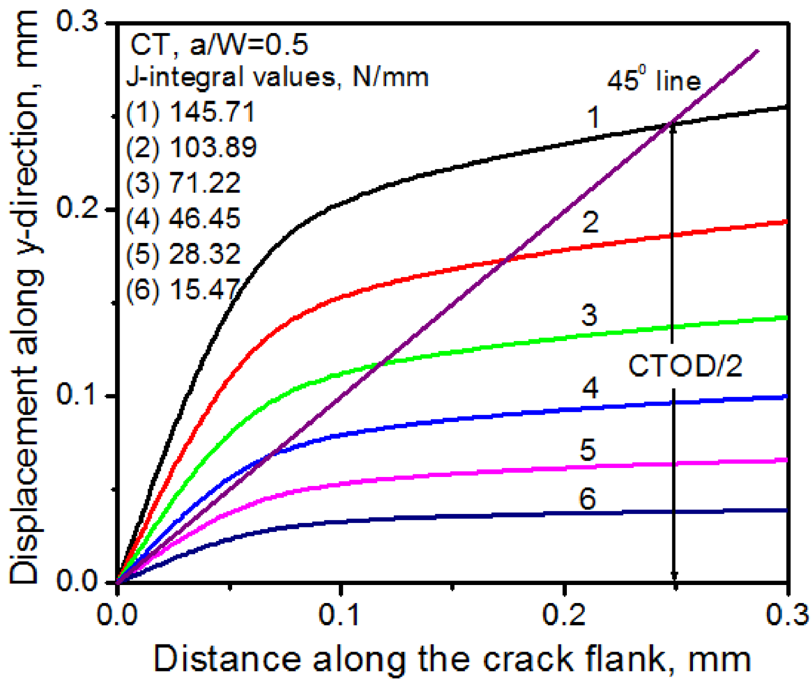

Figure 4: A typical plot of estimation of CTOD by 90° intercepts method for various magnitudes of J-integral.

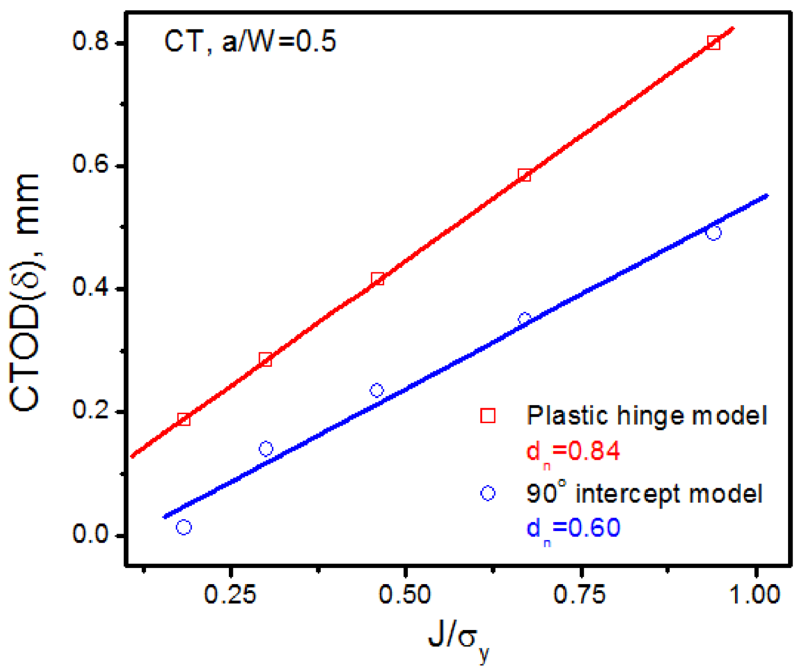

Figure 5: A typical plot variation of CTOD *vs.* J/σ_y for CT specimen.

crack mouth points with respect to a plastic hinge measured as a rotation factor (r) as given in Eq. 5. It is clearly mentioned in ASTM E1290, 99 [11] that the plastic rotation factor r is not a constant factor. The parameter r is a complex function of specimen configuration and size, applied loading and material. Recently, in the investigation of Kudari [15] it is shown that the rotation factor for particular specimen geometry is not a constant value; this differs with applied load and size of plastic zone ahead of a crack-tip. These difficulties in the measuring methods possibly alter the results of δ obtained by both the methods.

In the present investigation the constant d_n in the relationship between δ and J/σ_y (Refer Eq.(2)) is obtained by the slopes of the results shown in Fig.5 and Fig.6. The estimated values of constant, dn are 0.60 and 0.5 by 90° intercept method and 0.84 and 0.54 by plastic hinge model (calculated using Eq.5) for CT and SENB specimens respectively. These values are similar to FEA results of Shih [5] and Omidvar et al. [16]. This analysis demonstrates that there is a discrepancy in methods of measurements of CTOD and the value of constant d_n for analysis of thin sheets (considered as fully plastic case) is less than 1. It is also clear from this investigation that the conversion of J to δ may be associated with some errors depending on the magnitude of d_n and the method of estimation of CTOD. The analysis infers that while converting magnitude of δ to J one need to carefully evaluate the value of d_n depending on the material property, specimen geometry and method of estimation of δ rather than considering it to be 1.

The new relations between J and δ for the investigated IF steel (R-O constant N=3.358) based on method of estimation of CTOD for a CT and SENB specimen having *a/W*=0.5 are:

$$(\delta)_{PH} = 0.84\frac{J}{\sigma_y} \qquad (\delta)_{90^0} = 0.6\frac{J}{\sigma_y} \qquad \text{for CT} \quad (6)$$

$$(\delta)_{PH} = 0.54\frac{J}{\sigma_y} \qquad (\delta)_{90^0} = 0.5\frac{J}{\sigma_y} \qquad \text{for SENB} \quad (7)$$

where $(\delta)_{PH}$ and $(\delta)_{90°}$ are CTOD values estimated by plastic hinge model and 90° intercept method respectively. Using the above Eq. 6 and Eq. 7 one can find for a particular J/σ_y the magnitude of $(\delta)_{PH}$ is 28.57% higher than $(\delta)_{90°}$ for CT specimen and $(\delta)_{PH}$ is 7.40 % higher than $(\delta)_{90°}$ for SENB specimen. This result shows that there is inconsistency in relation between J and CTOD. This discrepancy in conversion of J and CTOD is found to strongly depend on the method of computation of δ and specimen geometry. It is also clear that the inconsistency in J and CTOD estimated in CT specimen is about four time that of SENB specimen.

In this investigation a possible effect of specimen *a/W* ratio (where, *a*-crack length and *W*- width of a specimen) on the magnitude of d_n is also studied. The magnitudes of the constant d_n have been estimated for specimens having *a/W*= 0.4, 0.45, 0.5, 0.55 and 0.6 in the similar manner as it is carried out for *a/W*=0.5. The effect of *a/W* ratio on the magnitude of estimated d_n is illustrated in Fig.7. This figure clearly shows that the magnitude of d_n is specimen *a/W* dependent, and is observed to be sensitive for CT specimen with *a/W*

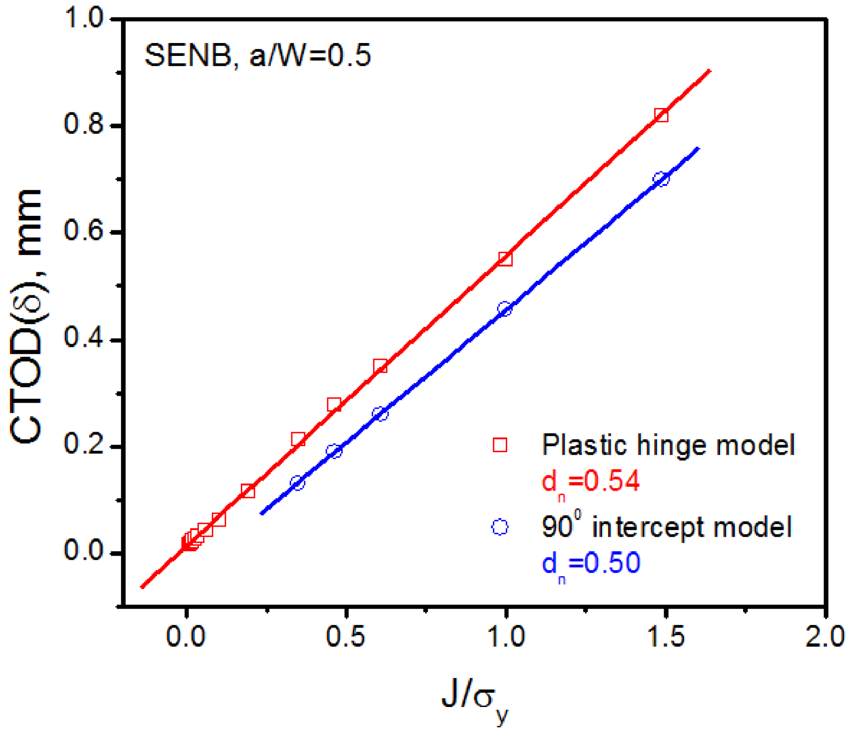

Figure 6: A typical plot variation of CTOD *vs.* J/σ_y for SENB specimen.

ratios <0.5. It also indicates that there is considerable difference in magnitudes of d_n obtained by both the methods of estimation of CTOD. The results in the Fig.7 shows that for CT specimens with *a/W*<0.50 the magnitudes of d_n are higher, and is found to be >1 if plastic hinge model is used. For CT specimens with *a/W* ≥ 0.50 the magnitudes of d_n are <1 and are almost constant. On the other hand, the SENB specimens show gradual increase in d_n as *a/W* increases from 0.4 to 0.6. It is also clear from Fig.7 that the discrepancy in magnitude of d_n in case of SENB specimen estimated by plastic hinge model and 90° intercept method decreases as *a/W* ratio increases from 0.4 to 0.6. The present results infer that the relationship between *J* and CTOD strongly depends on the method of estimation of CTOD, specimen geometry and *a/W* ratio of the specimens.

4. CONCLUSIONS

The following conclusions are drawn from the present investigation:

(i) There exists a discrepancy in estimation of δ from 90°-intercept method and by plastic hinge model.

(ii) The relationship between *J* and δ is linear and the linear proportionality constant, d_n, obtained in this analysis for a CT and SENB specimens with *a/W*=0.5 is found to be less than 1.

(iii) The relation between *J* and δ strongly depends on the method of estimation of, crack-tip opening displacement, specimen geometry and *a/W* ratio of the specimen.

5. ACKNOWLEDGEMENTS

Authors gratefully acknowledge the computational facilities provided by Research Center, B.V.B. College of Engineering and Technology, Hubli, India.

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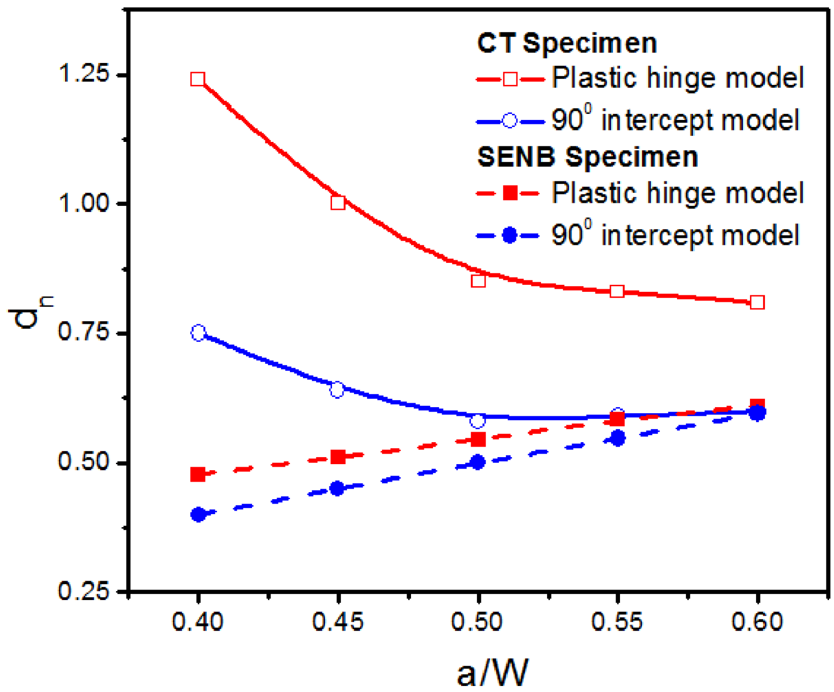

Figure 7: Effect of specimen *a/W* ratio on the magnitude of estimated $\boldsymbol{d_n}$.

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