Probabilistic prediction of fatigue damage based on linear fracture mechanics

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ABSTRACT. Paper describes in detail and gives example of the probabilistic assessment of a steel structural element subject to fatigue load, particular attention being paid to cracks from the edge and those from surface. Fatigue crack damage depends on a number of stress range cycles. Three sizes are important for the characteristics of the propagation of fatigue cracks - the initial size, detectable size and acceptable size. The theoretical model of fatigue crack progression in paper is based on a linear fracture mechanics. When determining the required degree of reliability, it is possible to specify the time of the first inspection of the construction which will focus on the fatigue damage. Using a conditional probability, times for subsequent inspections can be determined. For probabilistic calculation of fatigue crack progression was used the original and new probabilistic methods - the Direct Optimized Probabilistic Calculation ("DOProC"), which is based on optimized numerical integration. The algorithm of the probabilistic calculation was applied in the FCProbCalc code ("Fatigue Crack Probabilistic Calculation"), using which is possible to carry out the probabilistic modelling of propagation of fatigue cracks in a user friendly environment very effectively.

KEYWORDS. Direct Optimized Probabilistic Calculation; DOProC; Fatigue crack; Linear fracture mechanics; Probability of failure; Inspection of structure.

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INTRODUCTION

Numerous numerical methods, mostly based on the *finite element method* (FEM), have been developed to aid in the understanding of the behaviour of the fatigue phenomena, e.g. [7, 31]. Even though the use of *S-N* curves is well established in the design of structures, information relating to time-variable load and particularly to the detection of cracks from measurement made during the operational period of the bridge cannot be incorporated into reliability calculations [2, 24]. The essential tools for these calculations are provided by fracture mechanics and the reliability theory used in a probabilistic framework, e.g. [11, 13]. *Linear elastic fracture mechanics* – LEFM, is fully sufficient for fatigue crack propagation (LEFM is demonstrated to be a powerful tool to facilitate fatigue assessment due to the fact that initial cracks in real structures are unavoidable) on the condition of small scale yielding before the crack tip [6]. The initiation position of a fatigue crack is often found at an inclusion, impurity or surface flaw, which acts as a local stress raiser resulting in small scale plastic deformation. Analysis of the fatigue life based on linear elastic fracture mechanics requires that the random nature of production, crack growth, applied load and the subsequent failure due to cracking are properly taken into account. A prerequisite, however, is sufficient database of input random variables [4, 5 and 33].

Common studies of the misalignment of *failure probability* P_f (or the *reliability index* β) over time tend to focus only on structural details, however, a comprehensive probabilistic methodology generally applicable to bridge structures is currently missing. Insight into the stochastic interactions among random factors (load, geometric and material characteristics [8, 25, 27 and 28]) affecting the reliability of steel bridges is absolutely crucial to understanding the misalignment of failure probability of steel bridges [1]. Numerous stochastic methods, which enable the determination of failure probability, respectively the reliability index, have been developed [16, 22]. A substantial part of these methods are based on the crude *Monte Carlo* simulation method (MC), whose disadvantage is poor efficiency due to the need of a high number of simulation steps [26]. Advanced and stratified simulation methods, for e.g. *Markov chain Monte Carlo* simulations (MCMC), also applied in fatigue damage prognosis, strive to increase the efficiency of these computational methods [35]. An alternative solution is the evaluation of reliability using the reliability index, which can be efficiently determined using the *Latin Hypercube Sampling* method (LHS). The LHS method is capable of evaluating the reliability index from a small number of simulation steps (hundreds to thousands) [12]. Stochastic methods denoted as approximation methods enable probabilistic assessment of reliability analytically.

Another category of stochastic approaches for the quantification of the reliability of structures are represented by methods, which determine the failure probability on the basis of numerical integration, for e.g. the *Direct Optimized Probabilistic Calculation method* – DOProC, which was comprehensively published, e.g. in [9, 20]. It appears that this method is very effective for many probabilistic problems. It is also distinguished by higher accuracy than that of simulation methods, resp. approximation ones. This calculation method was applied in the solution of some engineering problems, among others, the assessment of reliability of steel bridge structures loaded by fatigue [19]. Probabilistic modelling of fatigue crack progression leads to designing a *system of regular inspections of structures* [21, 23].

MODELLING OF THE FATIGUE CRACK PROPAGATION USING LINEAR FRACTURE MECHANICS

Three sizes are important for the characteristics of the propagation of fatigue cracks. The first size is the *initiation size of the crack* that corresponds to a random failure in an element subject to random loads. Existence of the initiation cracks during the propagation should be revealed, along the *measurable length of the crack*, during inspections. The third important size has been referred to as far as the critical size – it is the final recorded size before a brittle fracture results in a failure. It would be advisable to use another method to specify the acceptable final size. Building structures and bridges are sized for extreme loads. Fatigue loads are investigated into only in details that are liable to fatigue cracks caused by variable operation loads [37]. If the load-bearing element is designed with a reasonable designed reliability margin for effects of the extreme load, then a crack will negatively influence the designed condition.

The fatigue crack damage depends on a number of stress range cycles. This is a time factor of reliability in the course of reliability for the entire designed service life [15, 36]. It is assumed that in the course of time the failure rate increases, while the reliability drops. If the propagation of the fatigue crack is included into the failure rate, it is necessary to investigate into the fatigue crack and define the maximum acceptable weakening. The weakening depends on the acceptable crack size which comprises safety margins for the critical crack size that may occur in consequence of a brittle fracture and, more often in steel structures, in consequence of a ductile fracture. The reason for this type of degradation of a load-bearing element in the course of time is the random existence of the initiation crack and propagation of the crack in the consequence of variable load effects. The result is the weakening of the element that has been sized for extreme load effects. The crack propagates in a stable way until it reaches the acceptable size that is a limit for the required reliability [14].

When investigating into the propagation, the fatigue crack that deteriorates a certain area of the structure components is described with one dimension only *a*. In order to describe the propagation of the crack, the linear elastic fracture mechanics is typically used. This method defines the limit of propagation rate of the crack (da/dN) and stress intensity factor range in the face of the crack using the Paris-Erdogan law [3, 30]:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C.\Delta K^m \tag{1}$$

where C, m are material constants, that are determined experimentally [32], a is the crack size, N is the number of loading cycles and ΔK is the stress intensity factor range. The fatigue crack will propagate in a stable way only if the initial crack a_0 exists in the place where the stress is concentrated. This place is located, e.g., at the edge or on the surface of the element. When using (1), the condition for the acceptable crack length a_{ac} is determined by:

$$N = \frac{1}{C}\int_{a_0}^{a_{ac}} \frac{\mathrm{d}a}{\Delta K^m} > N_{tot} \tag{2}$$

where N is the number of cycles needed to increase the crack from the initiation size a_0 to the acceptable crack size a_{ac}, and N_{tot} is the number of cycles throughout the service life. The Eq. (2) cannot be used, because the initiation crack size is not known.
The equation for the propagation of the crack size (1) needs to be modified for this purpose. If the stress range $\Delta\sigma$ is known, the range of the stress intensity factor range ΔK is:

$$\Delta K = \Delta\sigma.\sqrt{\pi.a}.F_{(a)} \tag{3}$$

where $F_{(a)}$ is the calibration function which represents the course of propagation of the crack. After the change of the number of cycles from N_1 to N_2, the crack will propagate from the length a_1 to a_2. Having modified (1) and using (3), the following formula will be achieved:

$$\int_{a_1}^{a_2} \frac{\mathrm{d}a}{\left(\sqrt{\pi.a}.F_{(a)}\right)^m} = \int_{N_1}^{N_2} C.\Delta\sigma^m.\mathrm{d}N \tag{4}$$

A tension flange has been chosen for applications of the theoretical solution suggested in the studies [34]. Depending on location of an initial crack, the crack may propagate from the edge or from the surface (see Figs. 1 and 2). Regarding the frequency, weight and stress concentration, those locations rank among those with the major hazard of fatigue cracks appearing in the steel structures and bridges.
A flange without stress concentration is used for confronting the both cases depending on the location of the crack initiation. The cases are different in calibration functions $F_{(a)}$ - and in weakened surfaces which are appearing during the crack propagation.

Fatigue cracks propagating from the edge

For the crack propagating from the edge, the calibration function is:

$$F_{(a)} = 1.12 - 1.36\cdot\frac{a}{b_f} + 7.32\cdot\left(\frac{a}{b_f}\right)^2 - 13.8\cdot\left(\frac{a}{b_f}\right)^3 + 14.0\cdot\left(\frac{a}{b_f}\right)^4 \tag{5}$$

where a is the length of the fatigue crack and b_f is the width of the flange (see Fig. 1). The acceptable crack size a_{ac} can be described then by a formula resulting from the deduced weakening of the cross-section area of the flange:

$$a_{ac} = b_f\cdot\left(1 - \frac{\sigma_{\max}}{f_y}\right) \tag{6}$$

where $\sigma_{\max}$ is maximal normal stress in the flange and f_y is yield stress of the steel.

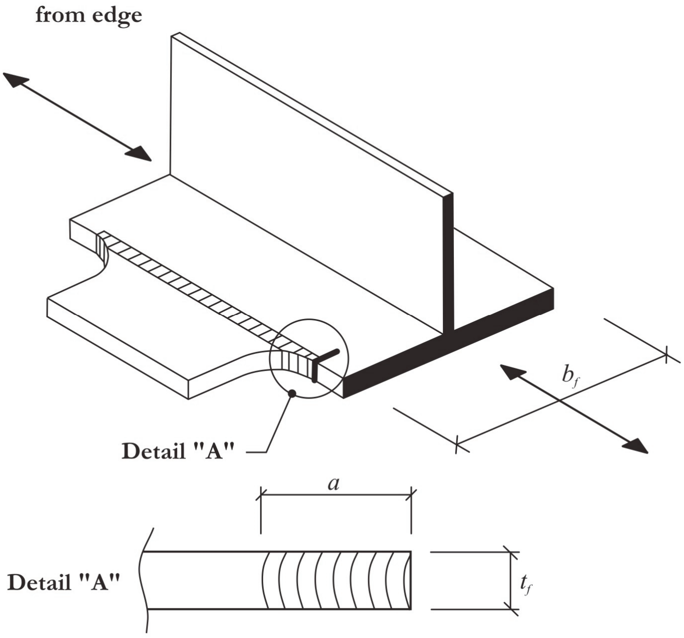

Figure 1: Detail of a fatigue crack in a flange plate subjected to tension (edge propagation).

Fatigue cracks propagating from the surface

A similar approach can be used to determine the acceptable size of a crack propagating from the surface. The bending component can be neglected for welded steel two-axis symmetric I-profiles where the fatigue crack appears in the lower tension flange. The flange is loaded only by the normal stress resulting from the axial load.

It is rather difficult to deduce analytically the acceptable size of the crack propagating from the surface. In accordance with [10], the shape is replaced with a semi-elliptic curve where the ellipsis axes are *a* (the crack depth) and *c* (a half of the crack width) - see Fig. 2. The area of the surface crack depends on the number of *N* loading cycles and is described by the following formula:

$$A_{cr(N)} = \frac{1}{2} \cdot \pi \cdot a_{(N)} \cdot c_{(N)} \tag{7}$$

During propagation of the fatigue crack from the surface, it is not enough to monitor only one crack size (which would be sufficient, for instance, for a crack propagating from the edge). In that case, the crack size needs to be analyzed for directions of the both semi-axes: *a* and *c*. The propagation of the fatigue crack from the surface in the *a* direction depends on the propagation in the *c* direction. Crack velocity propagation is described by Eq. (1). In [18] there is a formula for calculation of the crack depth $\Delta a_{(N)}$ as a result of an increased width of the $\Delta c_{(N)}$ crack:

$$\Delta a_{(N)} = \left\{ \frac{1}{\left[1.1 + 0.35 \cdot \left(\frac{a_{(N)}}{t_f} \right)^2 \cdot \sqrt{\frac{a_{(N)}}{c_{(N)}}} \right]} \right\}^m \cdot \Delta c_{(N)} \tag{8}$$

where t_f is the flange thickness.

The crack sizes for $a_{(N)}$ and $c_{(N)}$ are during the propagation limited by upper limit values:

$$2 \cdot c_{(N)} \leq 0.4 \cdot b_f \text{ and } a_{(N)} \leq 0.8 \cdot t_f \tag{9}$$

If these upper limit values are exceeded, the fatigue crack propagates differently. Publication [18] gives also the formula for the mutual dependence of the sizes in *a* and *c* :

$$c_{(N)} = 0.3027 \cdot \frac{a_{(N)}^2}{t_f} + 1.0202 \cdot a_{(N)} + 0.00699 \cdot t_f \tag{10}$$

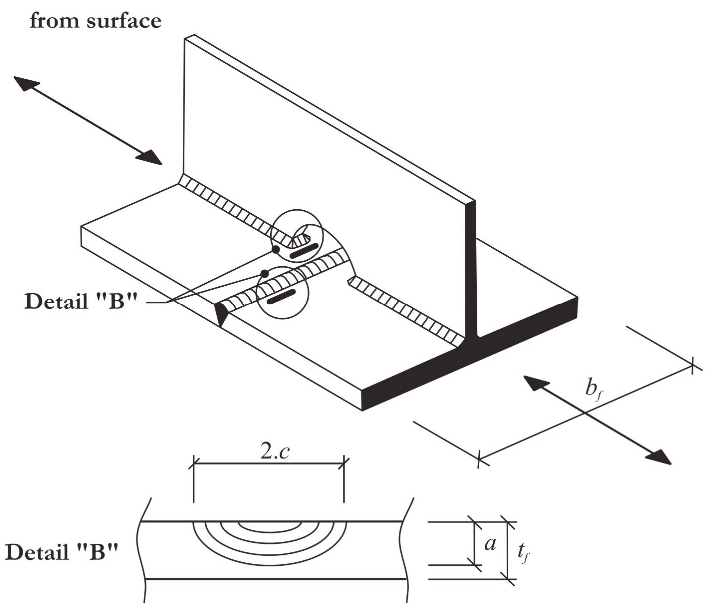

Figure 2: Detail of a fatigue crack in a flange plate subjected to tension (surface propagation).

When determining the acceptable crack size, a modified relation (7) using Eqs. (8) and (10), should be taken as a basis. After modification:

$$\sigma_{max} \cdot \frac{b_f \cdot t_f}{b_f \cdot t_f - \frac{1}{2} \cdot \pi \cdot a_{(N)} \cdot \left(0.3027 \cdot \frac{a_{(N)}^2}{t_f} + 1.0202 \cdot a_{(N)} + 0.00699 \cdot t_f \right)} \leq f_y \tag{11}$$

It is difficult to describe the $a_{(N)}$ crack size directly explicitly. In order to calculate the acceptable crack size a_{ac}, it is necessary to use a numerical iteration approach where restrictions resulting from Eq. (11) should be taken as a basis.

PROBABILISTIC RELIABILITY ASSESSMENT

The primary assumption is that the primary design should take into account the effects of the extreme loading and the fatigue resistance should be assessed then. This means, the reliability margin in the technical probability method is defined by:

$$g(R, E) = G = R - E \tag{12}$$

where *R* is the random resistance of the element and *E* represents random variable effects of the extreme load. If such element is subject to the operating load, following cases can occur:

a) **safe service life -** the fatigue effects do not degrade the element by means of the fatigue crack,
b) **acceptable failure rate** - the fatigue effects degrade the element and decrease the load-bearing capacity of the element,

c) **acceptable failure rate** - fatigue effects are expressed as stress changes.

The calculation model of the fatigue crack propagation defines the stress when the maximum acceptable crack results in the constant resistance of the structure, *R*, that corresponds to the stress in the yield point f_y. The approach c) is more demonstrative and has been preferred to the approach b) because it describes the non-linear growth of the both stresses in the element under degradation.

The probabilistic methods should be used for the investigation into the propagation rate of the fatigue crack until the *acceptable size* is reached because the input variables include uncertainties and reliability should be taken into account. The most important inputs are the initiation crack size and the acceptable crack size. The definition of the acceptable crack size/index is a necessary, but not the only one, condition because the initiation crack size is most important for the crack propagation. If the length of the crack a_1 equals to the initial length a_0 (this is the assumed size of the initiation crack in the probabilistic approach) and if a_2 equals to the final acceptable crack length a_{ac} (this is the acceptable crack size which replaces the critical crack size a_{cr} if the crack results in a brittle fracture), the left-hand side of Eq. (4) can be regarded as the resistance of the structure - *R*:

$$R_{(a_{ac})} = \int_{a_0}^{a_{ac}} \frac{\mathrm{d}a}{\left(\sqrt{\pi . a} . F_{(a)}\right)^m} \tag{13}$$

Similarly, it is possible to define the cumulated effect of loads that is equal to the right side (randomly variable effects of the extreme load) of Eq. (4):

$$E_{(N)} = \int_{N_0}^{N} C.\Delta\sigma^m.\mathrm{d}N = C.\Delta\sigma^m.\left(N - N_0\right) \tag{14}$$

where N is the total number of oscillations of stress peaks ($\Delta\sigma$) for the change of the length from a_0 to a_{ac}, and N_0 is the number of oscillations in the time of initialisation of the fatigue crack (typically, the number of oscillations is zero).

It is possible to define a reliability function G_{fail} :

$$G_{fail(\mathbf{X})} = R_{(a_{ac})} - E_{(N)} \tag{15}$$

where **X** is a vector of random physical properties such as mechanical properties, geometry of the structure, load effects and dimensions of the fatigue crack.

The analysis of the reliability function (12) gives a failure probability P_f, which equals to:

$$P_f = P\left(G_{fail(\mathbf{X})} < 0\right) = P\left(R_{(a_{ac})} - E_{(N)} < 0\right) \tag{16}$$

DETERMINATION OF STRUCTURAL INSPECTIONS

Because it is not certain in the probabilistic calculation whether the initiation crack exists and what the initiation crack size is and because other inaccuracies influence the calculation of the crack propagation, a specialised inspection is necessary to check the size of the measureable crack in a specific period of time. The acceptable crack size influences the time of the inspection. If no fatigue cracks are found, the analysis of inspection results give conditional probability during occurrence.

While the fatigue crack is propagating, it is possible to define following random phenomena that are related to the growth of the fatigue crack and may occur in any time, *t*, during the service life of the structure.

Then:

a) $\boldsymbol{U_{(t)}}$ **phenomenon:** No fatigue crack failure has not been revealed within the *t*–time and the fatigue crack size $a_{(t)}$ has not reached the detectable crack size, a_d . This means:

$$a_{(t)} < a_d \tag{17}$$

b) ***D*(*t*) phenomenon:** a fatigue crack failure has been revealed within the *t*–time and the fatigue crack size $a_{(t)}$ is still below the acceptable crack size a_{ac}. This means:

$$a_d \le a_{(t)} < a_{ac} \tag{18}$$

c) ***F*(*t*) phenomenon:** a failure has been revealed within the *t*–time and the fatigue crack size $a_{(t)}$ has reached the acceptable crack size a_{ac}. This means:

$$a_{(t)} \ge a_{ac} \tag{19}$$

If the crack is not revealed within the *t*-time, this may mean that there is not any fatigue crack in the construction element. This might be also an initiative phase of nucleation of the fatigue crack (when a crack appears in the material) and this phenomenon is not taken into account in the fracture mechanics. Even if the fatigue crack is not revealed it is likely that it exists there but the fatigue crack size is so small that it cannot be detected under existing conditions.
Using the phenomena above, it is possible to define following probabilities:
The probability that the failure is not detected within the *t*-time, this means the probability that the fatigue crack size $a_{(t)}$ is below the measurable crack size a_d :

$$P\left(U_{(t)}\right) = P\left(a_{(t)} < a_d\right) \tag{20}$$

The probability that the failure detected within the *t*-time has the crack size $a_{(t)}$ that is less than the acceptable size a_{ac}:

$$P\left(D_{(t)}\right) = P\left(a_d \le a_{(t)} < a_{ac}\right) \tag{21}$$

The probability that the failure occurs within the *t*-time, this means the probability that the fatigue crack size $a_{(t)}$ reaches the acceptable size a_{ac}:

$$P\left(F_{(t)}\right) = P\left(a_{ac} \le a_{(t)}\right) \tag{22}$$

Those three phenomena cover the complete spectrum of phenomena that might occur in the *t*-time. This means:

$$P\left(U_{(t)}\right) + P\left(D_{(t)}\right) + P\left(F_{(t)}\right) = 1 \tag{23}$$

The probable course of the growth of the fatigue crack is shown in Fig. 3.

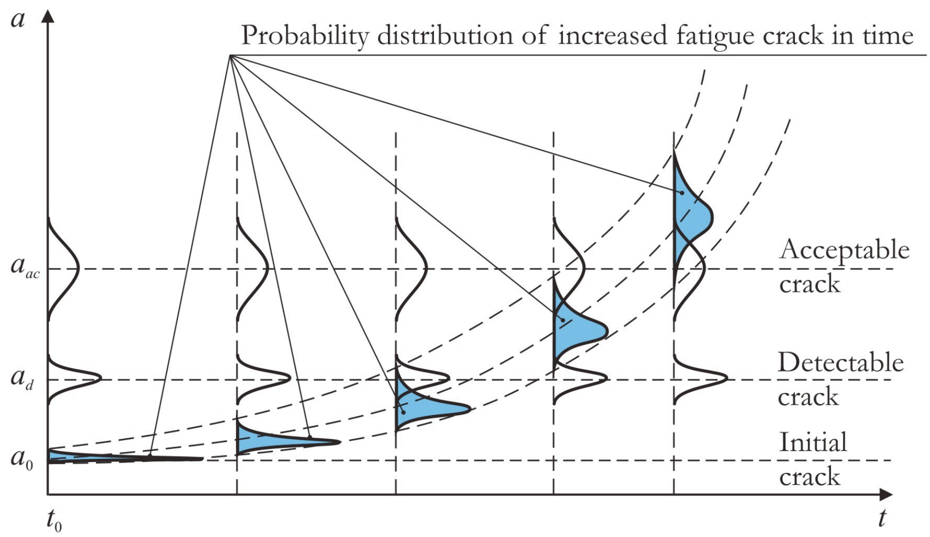

Figure 3: Probabilistic growth of the fatigue crack in the course of time *t*.

The probabilities in (20) through (22) can be determined in any period of time, t, using any of the probabilistic methods. The probabilistic calculation is carried out in time steps where one step typically equals to one year of the service life of the construction. When the probability of failure $P(F_{(t)})$reaches the designed failure probability P_d, an inspection should be carried out in order to find out fatigue cracks, if any, in the construction element. The inspection provides information about conditions of the construction. Such conditions can be taken into account when carrying out further probabilistic calculations. The inspection in the t time may result in any of the three mentioned phenomena.
Using the inspection results for the t time, it is possible to define the probaility of the mentioned phenomena in times $T > t_1$. For that purpose, the conditional probability should be taken into consideration (probability of A if B has occured):

$$P(A|B)=\frac{P(A\cap B)}{P(B)}\text{ , where } P(B)\neq 0 \tag{24}$$

The probability that the U phenomenon occurs in t_1 is:

$$\left.\begin{aligned}P(U_{(T)}|U_{(t_1)})&=\frac{P(U_{(T)}\cap U_{(t_1)})}{P(U_{(t_1)})}=\frac{P(U_{(T)})}{P(U_{(t_1)})}\\ P(D_{(T)}|U_{(t_1)})&=\frac{P(D_{(T)}\cap U_{(t_1)})}{P(U_{(t_1)})}\\ P(F_{(T)}|U_{(t_1)})&=\frac{P(F_{(T)}\cap U_{(t_1)})}{P(U_{(t_1)})}\end{aligned}\right\}\sum=1 \tag{25}$$

The probability that the D phenomenon occurs in t_1 is:

$$\left.\begin{aligned}P(U_{(T)}|D_{(t_1)})&=\frac{P(U_{(T)}\cap D_{(t_1)})}{P(D_{(t_1)})}=\frac{P(\{0\})}{P(D_{(t_1)})}=0\\ \left.\begin{aligned}P(D_{(T)}|D_{(t_1)})&=\frac{P(D_{(T)}\cap D_{(t_1)})}{P(D_{(t_1)})}\\ P(F_{(T)}|D_{(t_1)})&=\frac{P(F_{(T)}\cap D_{(t_1)})}{P(D_{(t_1)})}\end{aligned}\right\}&\sum=1\end{aligned}\right\}\sum=1 \tag{26}$$

The probability that the U phenomenon occurs in t_1 is:

$$\left.\begin{aligned}P(U_{(T)}|F_{(t_1)})&=\frac{P(U_{(T)}\cap F_{(t_1)})}{P(F_{(t_1)})}=\frac{P(\{0\})}{P(F_{(t_1)})}=0\\ P(D_{(T)}|F_{(t_1)})&=\frac{P(D_{(T)}\cap F_{(t_1)})}{P(F_{(t_1)})}=\frac{P(\{0\})}{P(F_{(t_1)})}=0\\ P(F_{(T)}|F_{(t_1)})&=\frac{P(F_{(T)}\cap F_{(t_1)})}{P(F_{(t_1)})}=\frac{P(F_{(t_1)})}{P(F_{(t_1)})}=1\end{aligned}\right\}\sum=1 \tag{27}$$

In order to specify the time for the next inspection, it is necessary to determine the conditioned probabilities which can be expressed using the full probability rule:

$$P(F_{(T)}|U_{(t_1)})=\frac{P(F_{(T)})-P(F_{(t_1)})-P(D_{(t_1)}).P(F_{(T)}|D_{(t_1)})}{P(U_{(t_1)})} \tag{28}$$

and

$$P\left(F_{(T)}\middle|D_{(t_I)}\right)=\frac{P\left(F_{(T)}\right)-P\left(F_{(t_I)}\right)-P\left(U_{(t_I)}\right).P\left(F_{(T)}\middle|U_{(t_I)}\right)}{P\left(D_{(t_I)}\right)} \tag{29}$$

If re-distribution of stress from a point that is weakened by the crack is not taken into account, the crack propagation crack is usually rather high in the practical range of detectable values. If a fatigue crack is found during the inspection, it is necessary to monitor the safe growth of the crack or to take actions that will slow down or stop further propagation of the fatigue crack. In order to time the inspections well, Eq. (28) is most important. It defines the failure probability in $T > t_I$ provided that no fatigue cracks have been revealed during the last inspection. It is clear from the equation that the results of the failure probability are influenced by mutual relations between the three crack sizes - the initial crack size a_0, measurable crack size a_d and acceptable crack size a_{ac}.
The probabilities in Eq. (28) can be calculated in any $T > t_I$ time using any probabilistic method. When the failure probability in Eq. (28) reaches the designed failure probability P_d, an inspection should be carried out in order to reveal fatigue cracks, if any, in the construction component. The inspection may result in one of the three mentioned phenomena with corresponding probabilities. The entire calculation can be repeated in order to ensure well-timed inspections in the future.

EXAMPLE OF PROBABILISTIC CALCULATION

The reference probabilistic calculation included the probabilistic assessment of a steel/reinforced concrete bridge from the highway [34] was performed in a detail, where a longitudinal steel beam connects to a steel transversal beam, which tends to suffer from fatigue damage. The input quantities were determined deterministically or stochastically using parametric probability distributions (see Tabs. 1 and 2). The required reliability was described by the reliability index $\beta = 2$ which corresponded to the designed probability of failure $P_d = 0.02277$.
Real input values were used in computation: the geometric shape in the specified detail, the yield stress f_y, the nominal designed stress of extreme impacts σ, material constants m and C, as well as range of stress oscillation $\Delta\sigma$. The source of the oscillation value was measurements of the response in regular operation. Other input data include the random quantities – they are expressed by means of the parametric distribution and were rather inaccurate if used as the input values. These values include the expected length of the detectable crack a_d, the number of load cycles per year N and, in particular, the size and exact location of the initiation crack a_0. Considering the detail of connection of the flange plate, it was decided to choose the mean value of $a_0 = 0.2$ mm with lognormal distribution. The chosen mean value $a_0 = 0.2$ mm with log-normal probability distribution represents a significant asymmetry histogram with a larger variance for $a_0 > 0.2$ mm according to [29].

Quantity	Value
Material constant m	3
Material constant C	$2.2{\cdot}10^{13}$ $\text{MPa}^{m}\text{m}^{(m/2)+1}$
Width of the flange plate b_f	400 mm
Thickness of the flange plate t_f	25 mm

Table 1: Overview of input deterministic quantities.

The probabilistic calculation was performed using FCProbCalc software ("Fatigue Crack Probability Calculation") [17], which has been developed using the aforementioned techniques. By means of FCProbCalc, it is possible to carry out the probabilistic modelling of propagation of fatigue cracks propagating from the edge and from the surface in a user friendly environment and to propose a system of regular inspections which should reveal damage to the structure.
If a period of time is specified and the time step is 1 year, it is possible to determine resistance of the construction $R_{(a_{ac})}$ pursuant to (13) (so far, five types of numerical integration are available; comparison their accuracy and efficiency in [18]), load effects, $E_{(N)}$, pursuant to (14), as well as the probability of elemental phenomena, U, D and F, pursuant to Eqs. (20) through (22) which are the basis for specification of inspection times.

Quantity	Type of parametric probability distribution	Mean value	Standard deviation
Range of stress peaks $\Delta\sigma$	Normal	30 MPa	3 MPa
Total number of stress peaks per year N	Normal	10^6	10^5
Yield stress f_y	Lognormal	280 MPa	28 MPa
Nominal stress in the flange plate σ	Normal	200 MPa	20 MPa
Initial size of the crack a_0	Lognormal	0.2 mm	0.05 mm
Smallest detectable size of the crack a_d	Normal	10 mm	0.6 mm

Table 2: Overview of input random quantities expressed in histograms with parametric distribution of probability.

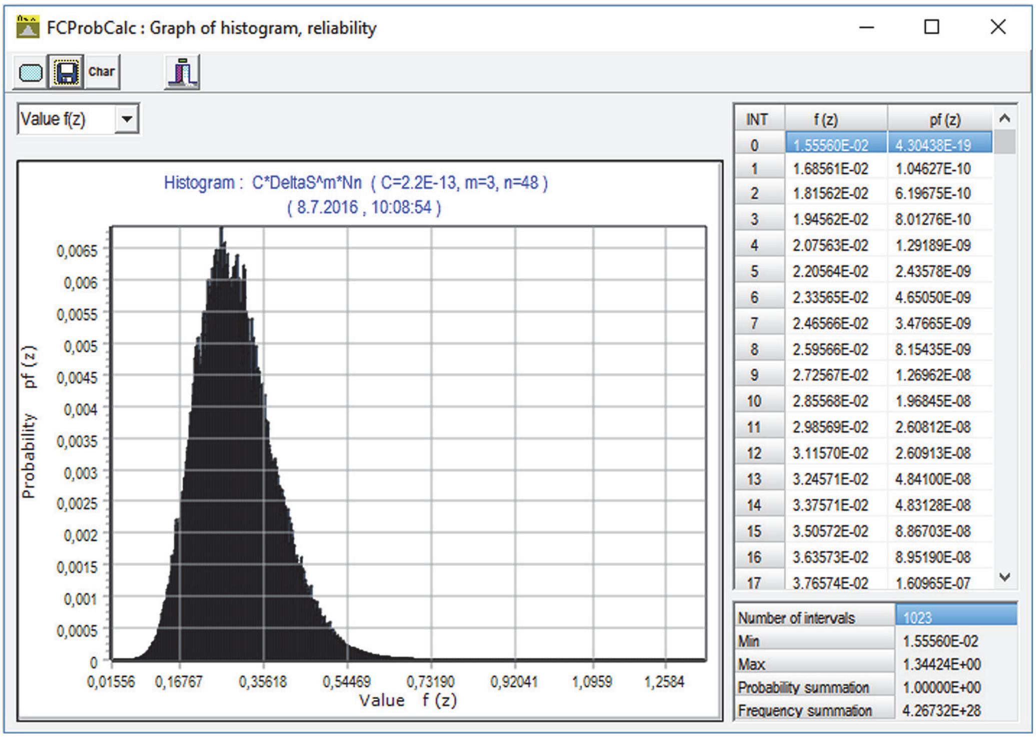

Figure 4: Resulting histogram for the $E_{(N)}$ load effects for a bridge structure after N = 48 years of operation.

Figs. 4 through 7 show results of the probabilistic modelling of a fatigue crack from the edge. Figs. 4 and 5 show resulting histograms during the first inspection for load effects, E, as well as resistance of the structure, $R_{(a_{ad})}$. Fig. 6 shows chart with calculated probabilities of the U, D and F events resulting from Eqs. (20) through (22) and taking into account Eq. 23. Fig. 7 show times for the first inspection and subsequent inspections resulting from the conditional probability pursuant to Eq. (28). The curves describe dependence of the probability of failure, P_f, on time of operation of the bridge structure. When the probability of failure exceeds the specified designed probability, P_d, the inspection should be performed. It was decided that the first inspection of the bridge should take place after 48 years of operation. This inspection will focus on growth of the fatigue crack on the edge. The Tab. 3 include a table with numerical values for the final inspection times.

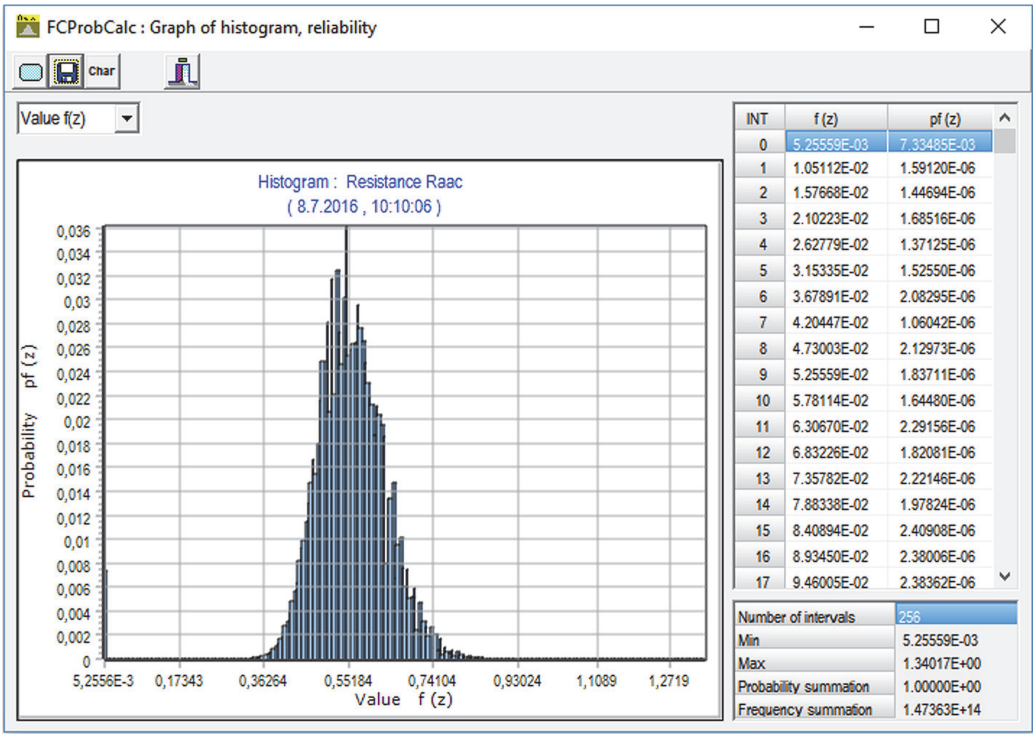

Figure 5: Resulting histogram for the resistance $R_{(a_{ac})}$ for the bridge structure focused to a **fatigue crack from the edge**, the 15-points Gaussian quadrature with numerical integration was used.

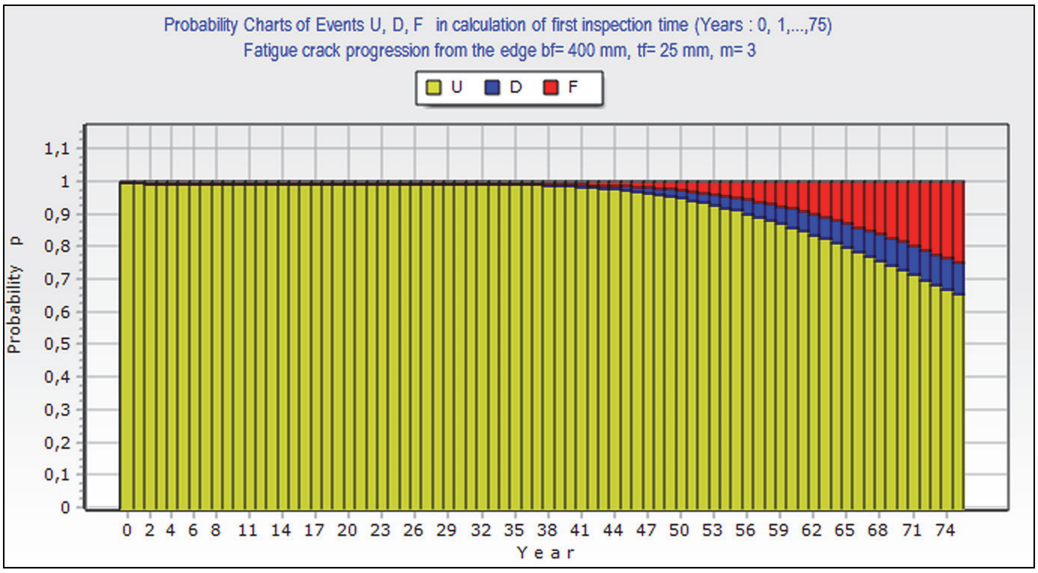

Figure 6: Calculated probabilities of elemental phenomena, U, D and F, in a bridge structure focused to a **fatigue crack from the edge** (N = 0 to 75 years), the 15-points Gaussian quadrature with numerical integration was used.

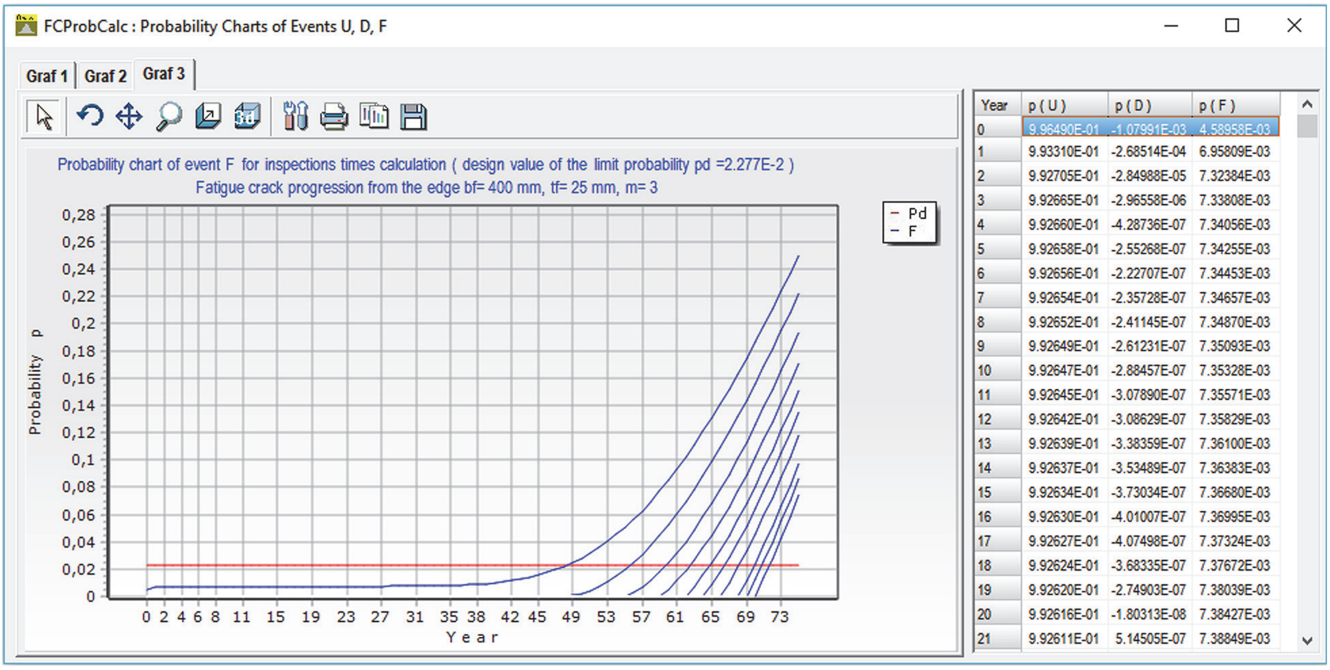

Figure 7: Dependence of the failure probability, P_f, on years of operation of the bridge structure during the probabilistic calculation of propagation of **fatigue cracks from the edge** (N = 0 to 75 years) with respect to the conditional probability and specification of the time for the first and subsequent inspections of the bridge structure, the 15-points Gaussian quadrature with numerical integration was used.

	Time of inspection [years]	
Inspection No.	Fatigue crack from the edge	Fatigue crack from the surface
1	**48**	**109**
2	55	122
3	59	130
4	62	136
5	64	141
6	66	145
7	68	149
8	69	152
9	70	155
10	71	158

Table 3: Calculated times for the first and subsequent inspection of the bridge structure - propagation of the fatigue cracks from the edge and the surface, the 15-point Gaussian quadrature with numerical integration was used.

Figs. 8 through 11 shows results of the probabilistic modelling of a fatigue crack from the surface.
In the study was analyse probabilistic calculation of fatigue crack progression from the surface also. It follows from the comparison of times for the first inspections which focus on the fatigue damage by the both types of the fatigue cracks (after 48 years of operation for the edge crack and after 109 years of operation for the surface crack) that the fatigue cracks propagate from the surface with a considerably lower speed that the fatigue cracks which initiate at the edge (see Tab. 3).

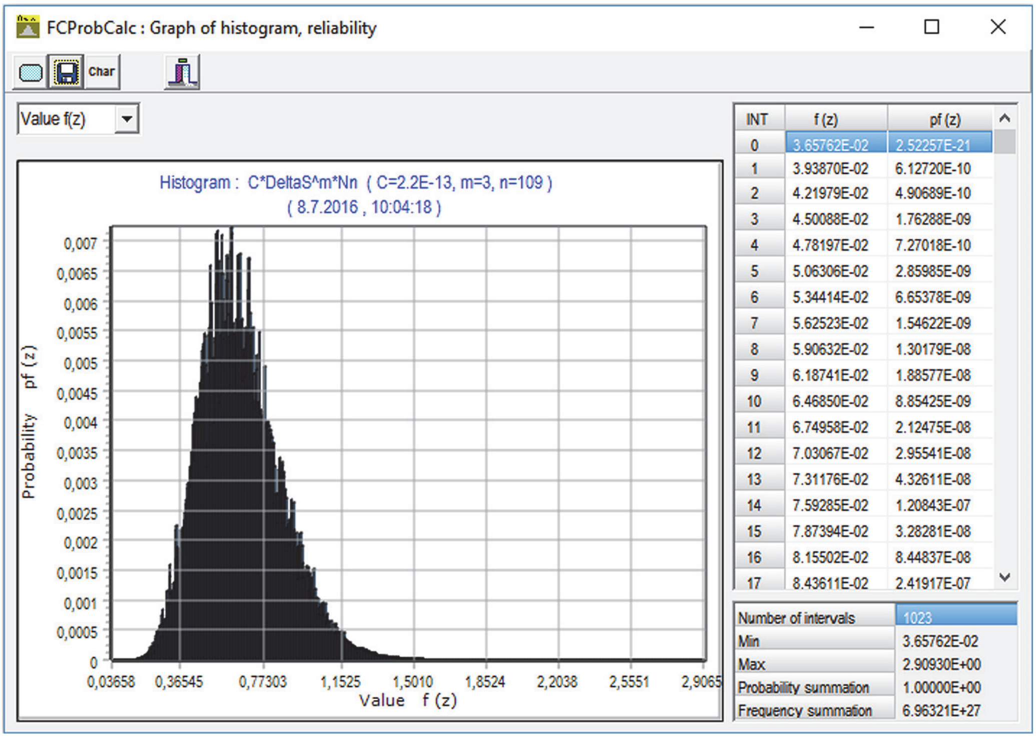

Figure 8: Resulting histogram for the $E_{(N)}$ load effects for a bridge structure during the first inspection after N = 109 years of operation.

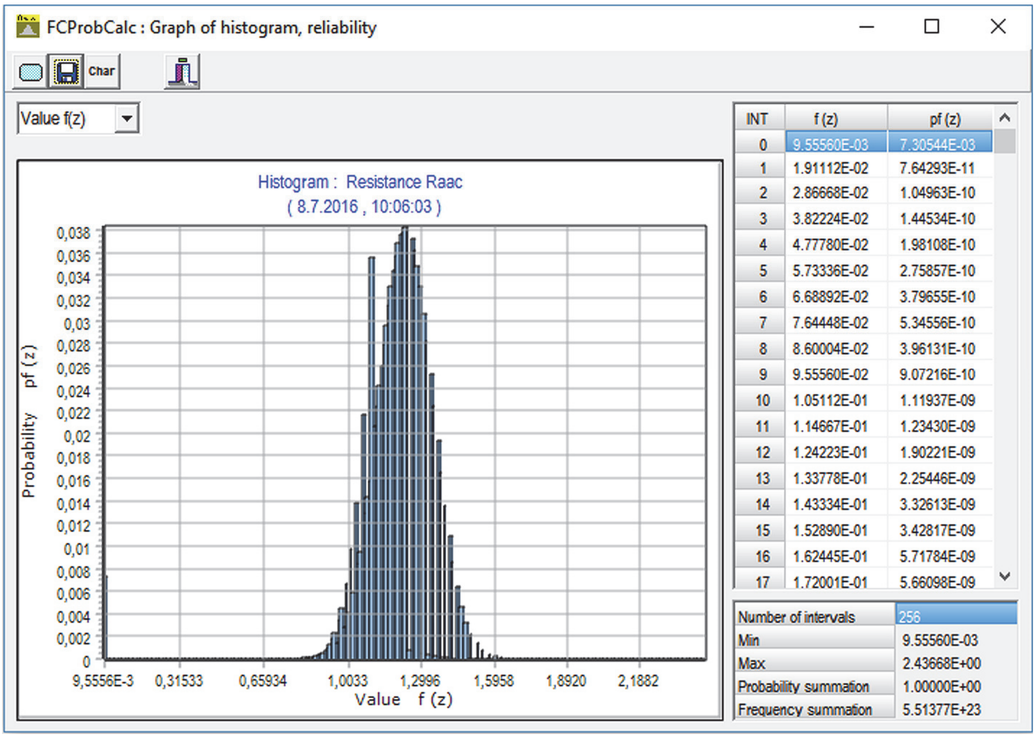

Figure 9: Resulting histogram for the resistance $R_{(a_{ac})}$ for the bridge structure focused to a **fatigue crack from the surface**, the 15-points Gaussian quadrature with numerical integration was used.

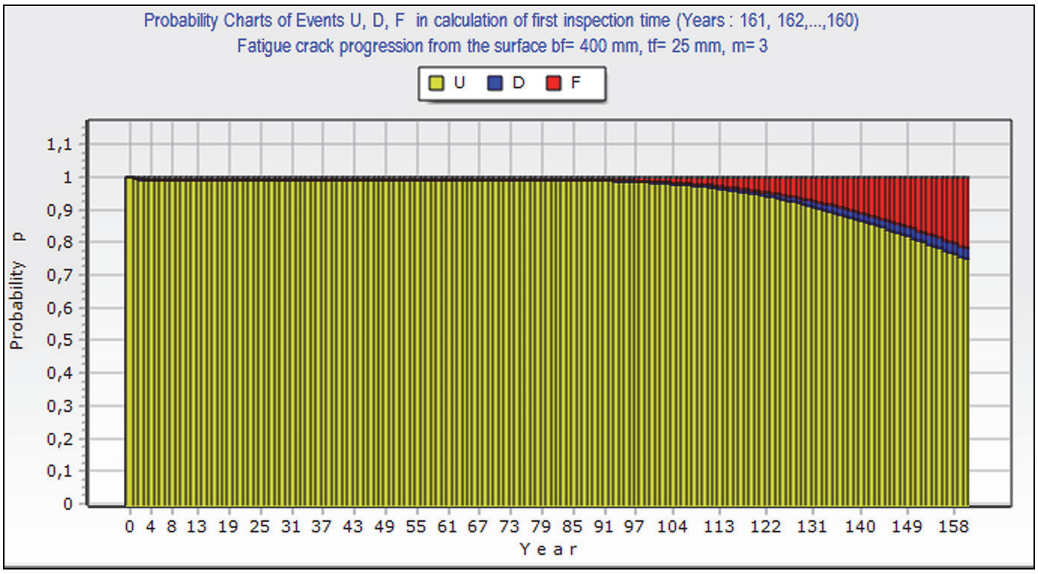

Figure 10: Calculated probabilities of elemental phenomena, *U*, *D* and *F*, in a bridge structure focused to a **fatigue crack from the surface** (N = 0 to 160 years), the 15-points Gaussian quadrature with numerical integration was used.

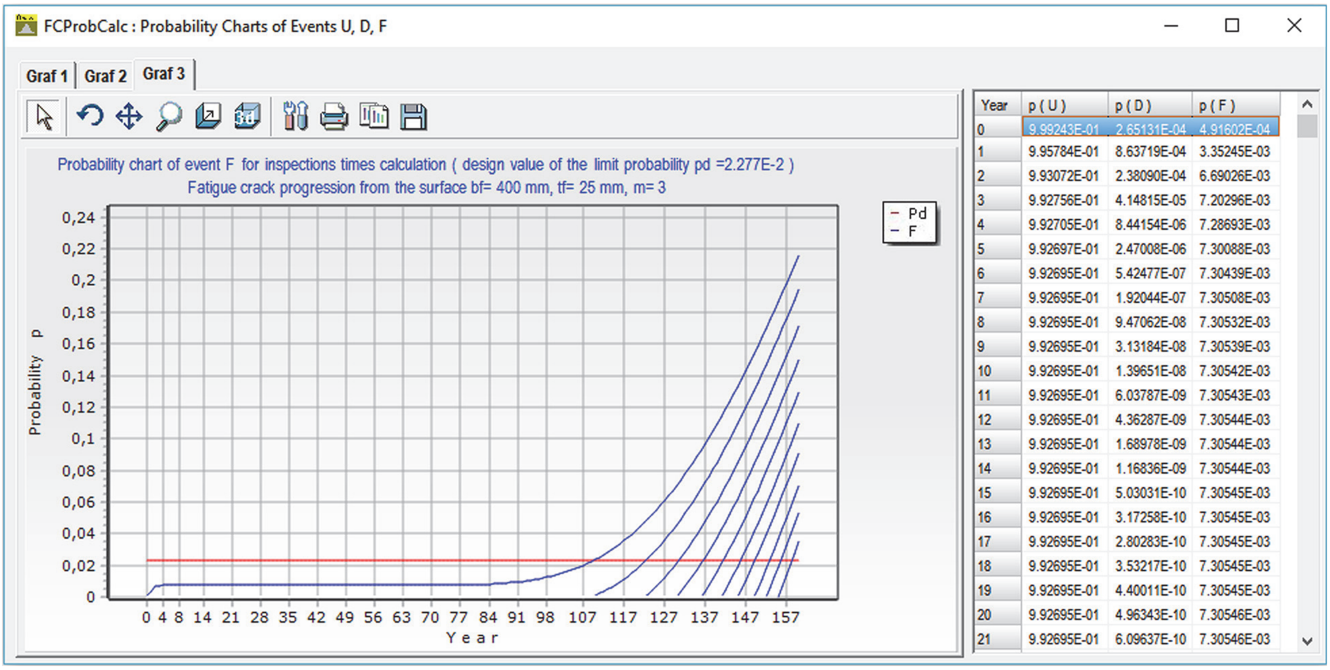

Figure 11: Dependence of the failure probability, P_f, on years of operation of the bridge structure during the probabilistic calculation of propagation of **fatigue cracks from the surface** (N = 0 to 160 years) with respect to the conditional probability and specification of the time for the first and subsequent inspections of the bridge structure, the 15-points Gaussian quadrature with numerical integration was used.

CONCLUSION

This paper describes methods dealing with propagation of fatigue cracks from the edge/surface in steel structures and bridges, which are subject to cyclic loads. A particular attention is paid to the maximum acceptable crack size. The theoretical model of fatigue crack progression is based on a linear fracture mechanics.

Propagation of the fatigue cracks and possible prediction of such propagation in time since the start of variable loading effects is the case when probabilistic methods must be used because too many uncertainties influence the determination of the input values. The uncertainties include both loading effects and construction resistance (for instance, the stochastic response to effects of the variable operation form by oscillation of stress in locations which are susceptible to fatigue damage). In the global context, it is the size of the expected initial crack, which is managed with most difficulties.
The calculation uses the newly developed Direct Optimized Probabilistic Calculation ("DOProC"), which is suitable for several probabilistic calculations. Examples of the probabilistic methods used in calculations have been proving that the method is suitable not only for the reliability assessment, but also for other probabilistic calculations, including the propagation of the fatigue cracks. DOProC appears to be a very efficient tool that results in the solution affected by a numerical error and by an error resulting from the discretizing of the input and output quantities only.
The probability of propagation of fatigue cracks from the edge/surface was calculated in FCProbCalc. Using this software, it was possible to make probabilistic assessment of the structural reliability on the basis of the exact definition of the acceptable size of the fatigue crack. The probabilities were obtained for three basic phenomena, which are related to propagation of the fatigue cracks. On the basis of those data, the probability of failure can be calculated for each year of operation of the construction. When determining the required degree of reliability, it is possible to specify the time of the first inspection of the structure, which will focus on the fatigue damage. Using a conditional probability, times for subsequent inspections can be determined.
The methods and application can considerably improve estimation of maintenance costs for the structures and bridges subject to cyclic loads. If this methodology is developed further, the goal of investigations seems to be, in particular, application of Bayesian networks in the computational model, which describes propagation of fatigue cracks.

APPENDIX

The application of mentioned DOProC approach have been implemented into FCProbCalc code. A light version of this software is available for downloading at http://www.fast.vsb.cz/popv.

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