



## Fatigue crack growth behavior under multiaxial variable amplitude loading

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**ABSTRACT.** This study compares both uniaxial and multiaxial variable amplitude experimental crack growth data for naturally initiated fatigue cracks in tubular specimens of 2024-T3 aluminum alloy to predictions based on two state-of-the-art analysis codes: UniGrow and FASTRAN. For variable amplitude fatigue tests performed under pure axial nominal loading conditions, both UniGrow and FASTRAN analyses were found to produce mostly conservative growth life predictions, despite good agreement with constant amplitude crack growth data. For variable amplitude torsion and combined axial-torsion crack growth analyses, however, the conservatism in growth life predictions was found to reduce. This was attributed to multiaxial nominal stress state effects, such as T-stress and mixed-mode crack growth, which are not accounted for in either UniGrow or FASTRAN, but were found in constant amplitude fatigue tests to increase experimental crack growth rates. Since cracks in this study were initiated naturally, different initial crack geometry assumptions were also investigated in the analyses.

**KEYWORDS.** Fatigue Crack Growth; Multiaxial; Variable Amplitude; FASTRAN; UniGrow.

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### INTRODUCTION

Most engineering components and structures are subjected to variable amplitude cyclic loadings throughout their service lives. Due to the nature of these loadings, they typically result in multiaxial stress states, and individual stress components can vary in a non-proportional manner. When such components are operating under a damage tolerant design philosophy, being able to predict how fatigue cracks will grow under these complex loading conditions is a topic of particular interest.

One of the key requirements for the application of fracture mechanics concepts in a fatigue crack growth analysis is that conditions of similitude should be retained. Similitude implies that for a particular value of driving force parameter (e.g. J-

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Integral, stress intensity factor (SIF), crack tip opening displacement (CTOD), etc.), the state of stress surrounding the crack tip is uniquely described by the value of that parameter. However, for variable amplitude loading, load history dependence may alter the local stress state at a crack tip allowing the possibility for multiple crack growth rates to occur at the same nominal driving force. Because of this, the amount of crack growth experienced in a variable amplitude loading history cannot be accurately assessed by simply summing the nominal crack growth increment for each applied cycle. Load sequence effects have been attributed to a number of mechanisms including: crack blunting, compressive residual stresses in front of the crack tip, changes in plasticity induced crack closure due to varying amounts of residual deformation in the crack wake, crack deflection (i.e. increased roughness induced closure), and strain hardening effects. Although all of these mechanisms may contribute in some degree, residual stresses and changes in closure levels tend to be the favored explanations [1].

In addition to load sequence effects on mode I crack growth under uniaxial nominal loading, multiaxial nominal stress states can also affect growth rates. For example, Gladskyi and Fatemi [2] studied axial and torsion load sequence effects on mode I crack growth and found that for cracks growing in mode I under pure torsion and pure axial loadings, cracks in pure torsion tests grew faster despite the same maximum principal stress range. Additionally, the insertion of blocks of pure torsion cycles into an otherwise uniaxial loading history was shown to increase crack growth rate, while the insertion of uniaxial cycles into a pure torsion loading history was shown to decrease crack growth rate. Most of these effects were explained in terms of the stress state at the crack tip. For mode I cracks growing under nominal torsion loading, it was speculated that the presence of a tangential stress (T-stress) component, acting parallel to the direction of crack growth, increased crack tip driving force for a given value of nominal SIF. Crack growth rate correlations for the various loading histories were found to improve when considering the effects of T-stress on plastic zone size.

In this study, crack growth data were generated under a variety of variable amplitude loading conditions using notched tubular specimens of 2024-T3 aluminum alloy. Given the complexity of such an analysis, two state-of-the-art analysis codes were used in order to compare the experimentally measured crack growth lives to predictions based on two fundamentally different crack growth models. UniGrow [3] is based on the idea that residual stress distributions surrounding the crack tip are responsible for causing load sequence effects in variable amplitude crack growth, while FASTRAN [4] attributes these effects to varying degrees of plasticity induced closure in the crack wake. Since both analysis programs are meant for application to crack growth under uniaxial loading conditions, variable amplitude crack growth trends for multiaxial nominal loadings are compared with those for constant amplitude loading conditions in order to help interpret the analysis results.

## MATERIAL AND TESTING PROCEDURES

The material chosen for all fatigue tests performed in this study was aluminum alloy 2024-T3. Tests were performed using notched specimens of a thin-walled tubular geometry. The specimens feature a 30 mm long gage section with an outside diameter of 29 mm and an inside diameter of 25.4 mm, resulting in a wall thickness of 1.8 mm. To serve as a stress concentrator, a 3.2 mm diameter circular transverse hole was produced using a drilling and reaming operation through one side of the specimen gage section. Material properties and complete specimen geometry can be found in [5].

All variable amplitude fatigue tests were based on a single stress-based simulated service loading history representing the nominal axial and shear loading conditions on the lower wing skin of a long-range military patrol aircraft. A variety of take-off, landing, and in-flight maneuvers are represented in the history. In its entirety, the loading history contains around 915000 data points, with each point approximately corresponding to one loading reversal on the axial stress channel. The maximum and minimum axial stresses in the unscaled history are 144.8 MPa and -51.3 MPa, respectively, while the maximum and minimum shear stresses are 67.0 MPa and -15.9 MPa, respectively. Plots showing the time history of a 1000 reversal segment taken from the loading history, along with the axial-shear stress path for this same segment, are shown in Fig. 1. The loading segment shown in this figure is representative of the loading patterns repeated throughout the remainder of the full history. From the stress path, it can be seen that the loading history contains significant non-proportional loading events. Different loading conditions were obtained in testing by using the axial loading channel only, the torsion channel only, or the combined axial-torsion loading.

Variable amplitude fatigue tests were performed using a closed loop servo-hydraulic axial-torsion load frame by repeatedly applying the entire load history in nominal load control until a tip-to-tip crack length of 15 mm was reached. For each test, the entire loading history was scaled by an appropriate factor to obtain stress levels that would produce fatigue lives ranging from less than one block to around 10 blocks. A summary of the applied loading conditions for all

tests can be found in [5]. In addition to the variable amplitude fatigue tests, a variety of constant amplitude fatigue tests were also performed on notched specimens under fully-reversed axial, torsion, and combined axial-torsion loading. Some results from these tests are presented herein as a basis for evaluating the variable amplitude crack growth predictions. Crack initiation and growth were monitored by using a digital microscope camera, capable of 10–230x optical zoom levels. Crack lengths were then measured using digital image analysis software.

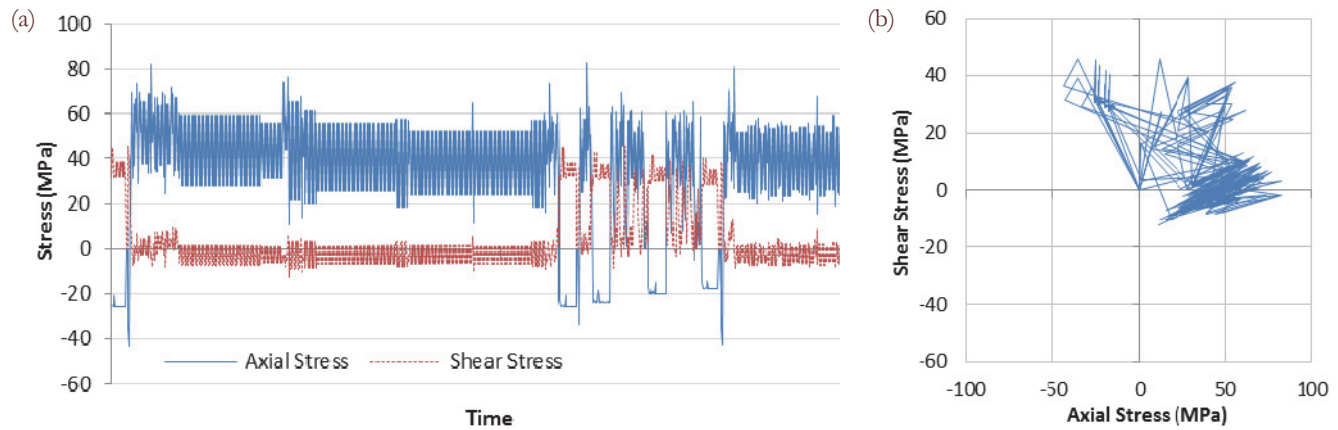


Figure 1: Representative loading segment from the variable amplitude service loading history in terms of (a) applied stresses vs. time and (b) axial-shear stress path.

## MODELING PROCEDURES

Because constant amplitude crack growth rate data for the specimens tested in this study were only generated under fully-reversed loading conditions, they are not ideal for computing crack growth properties for either the UniGrow or FASTRAN software. As a result, the UniGrow analyses performed in this study (version 2014-02-09) utilized material properties for a 2024-T351 aluminum alloy, which were already included in the UniGrow material library. Similarly, FASTRAN (version 5.42) analyses were based on crack growth properties reported in literature for 2024-T3 [6]. The mechanical and crack growth properties for the materials used in both programs were found to be very similar to those measured experimentally for the 2024-T3 alloy used in this study. Consequently, using the material data from literature is not expected to have a significant effect on the accuracy of crack growth predictions presented in the following section for either program.

Crack growth was only measured on the outer surface of the specimens tested in this study. Therefore, it is reasonable to assume that while cracks were short, their geometry was either that of a through thickness crack or corner crack growing from the edge of the hole. Additionally, since cracks were observed to grow from both sides of the hole, the assumption of diametrically opposite symmetric cracks was considered reasonable. As a result, two different specimen geometries were used in the following analyses: a circumferential through crack in a tube (TT), and two symmetric semi-elliptic corner cracks at a hole in a plate subjected to remote tensile stress (CCH). The CCH geometry was assumed to transition to the TT geometry after cracks became through thickness. The same SIF solutions were defined as custom inputs in each crack growth program.

Because stress intensity factors for short cracks are lower for the CCH geometry, the predicted crack growth lives are longer than those based on the TT crack geometry assumption. Since it is possible for actual cracks to assume either of these geometries, or a combination of both, crack growth predictions based on both TT and CCH SIF solutions should provide an approximate upper and lower bounds for experimentally observed crack growth curves, so long as the crack growth models are accurate.

For each nominal loading history, stresses were projected onto the maximum principal stress plane (consistent with observed crack growth planes) and input into the crack growth programs for analysis. This corresponds to the  $0^\circ$  plane (perpendicular to the specimen axis) for axial and combined variable amplitude loading histories, and the  $45^\circ$  plane for pure torsion loading. Crack growth lives were compared from an initial half crack length of 1.8 mm (0.2 mm excluding the hole radius) to a length of 7.5 mm.



## CRACK GROWTH PREDICTION RESULTS

In order to establish some baseline crack growth prediction results, both UniGrow and FASTRAN were used to analyze crack growth under simple constant amplitude loading conditions. These results, shown in Fig. 2 for the both TT and CCH crack geometries, are useful when interpreting the results of subsequent variable amplitude analyses.

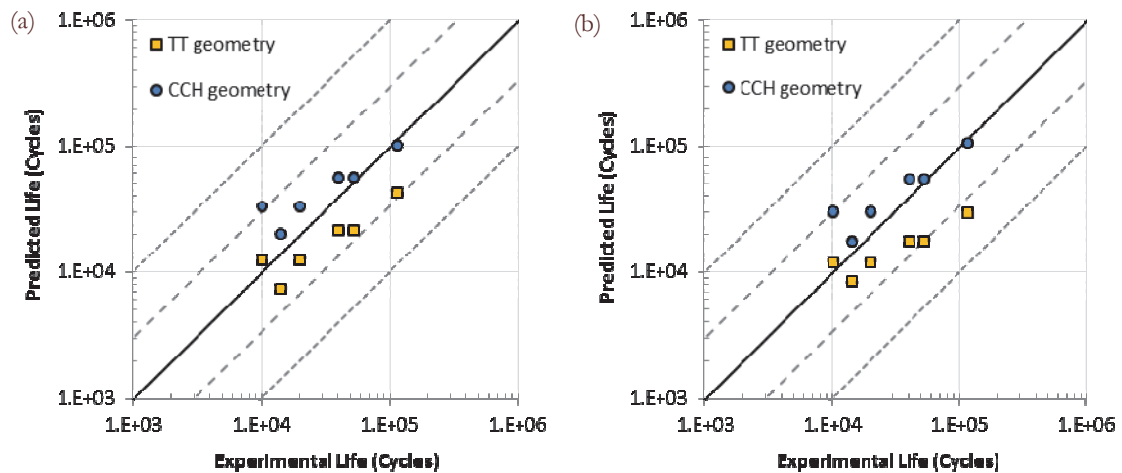


Figure 2: Constant amplitude crack growth life predictions based on (a) FASTRAN and (b) UniGrow analyses.

From these results, it is clear that both UniGrow and FASTRAN are capable of producing fairly accurate crack growth predictions under constant amplitude loading conditions. While UniGrow gives slightly more conservative crack growth predictions than FASTRAN, Fig. 2 shows that the majority of life predictions are still within a factor of  $\pm 3$  of experimental results, regardless of crack geometry assumption. Additionally, the fact that most experimental crack growth lives fall between the TT and CCH life predictions supports the idea that these predictions can be used to represent the lower and upper bounds for crack growth life, respectively.

The first variable amplitude loading conditions analyzed were those corresponding to the pure axial (A) nominal loading history. Plots of experimental versus predicted crack growth life for each analysis performed are shown in Fig. 3 for both crack geometries. Additionally, experimental versus predicted crack growth curves are shown in Fig. 4(a) for the lowest applied loading level. From these results, both UniGrow and FASTRAN are generally found to produce conservative crack growth predictions for the loading history utilized, regardless of the assumed crack geometry. This is despite the fact that, for both programs, experimental crack growth lives under constant amplitude loading conditions were generally found to fall between predictions based on the TT and CCH crack geometry assumptions. In general, the degree of conservatism in crack growth predictions tends to increase with decreasing stress levels.

By comparing these results to the constant amplitude baseline analyses, the effects of the variable amplitude loading history are found to be greater for life predictions based on the UniGrow crack growth model than for those based on FASTRAN. While FASTRAN growth life predictions for constant amplitude tests were, on average, a factor of 1.1 times longer than UniGrow predictions, this difference increased to an average of 1.8 times longer for variable amplitude tests. The fact that variable amplitude growth life predictions for both analysis programs are closer to the experimentally measured lives as the loading level increases suggests that increased plasticity is likely not the cause for the prediction error.

In addition to tests performed under axial only loading conditions, crack growth for the pure torsion (T) variable amplitude loading histories was also analyzed, shown in Fig. 3. From Figs. 3 and 4(b), crack growth predictions for the nominal torsion loading histories are found to be significantly different than those for axial loading. Instead of being consistently conservative, the torsion growth predictions tend to be non-conservative based on FASTRAN analyses, and fairly accurate based on UniGrow analyses. Additionally, the shift in conservatism is notably larger for the FASTRAN analyses than for the UniGrow analyses. While FASTRAN life predictions for the axial loading history were an average of 1.9 times longer than UniGrow predictions, this increases to a factor of 8.5 for the torsion loading history analyses. Based on the differences between crack growth conditions under the pure axial and pure torsion loading histories applied in this



study, the cause for these shifts in prediction accuracy can be narrowed down to two likely sources: multiaxial stress state effects, such as the presence of T-stress, and/or differences in loading history profile.

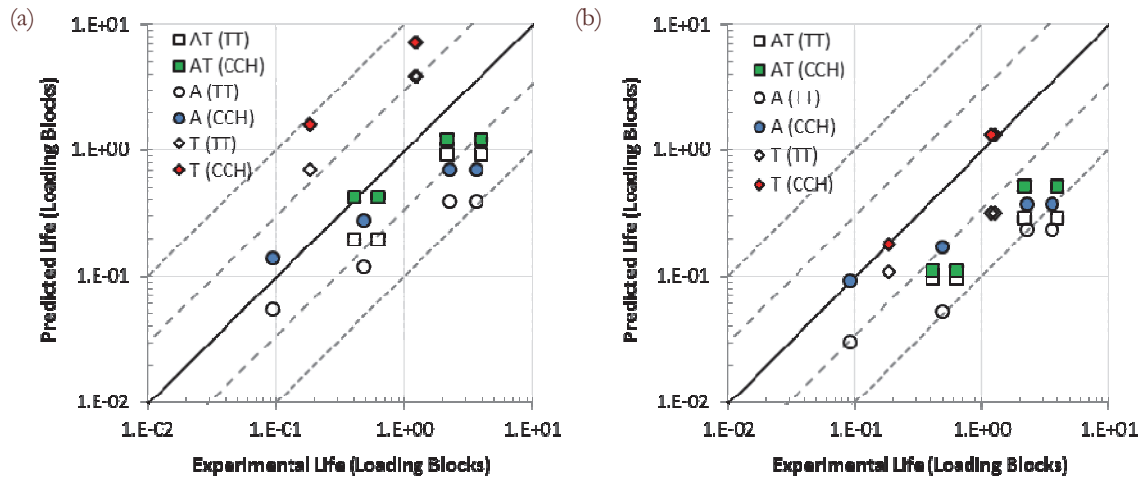


Figure 3: Variable amplitude crack growth life predictions based on (a) FASTRAN and (b) UniGrow analyses.

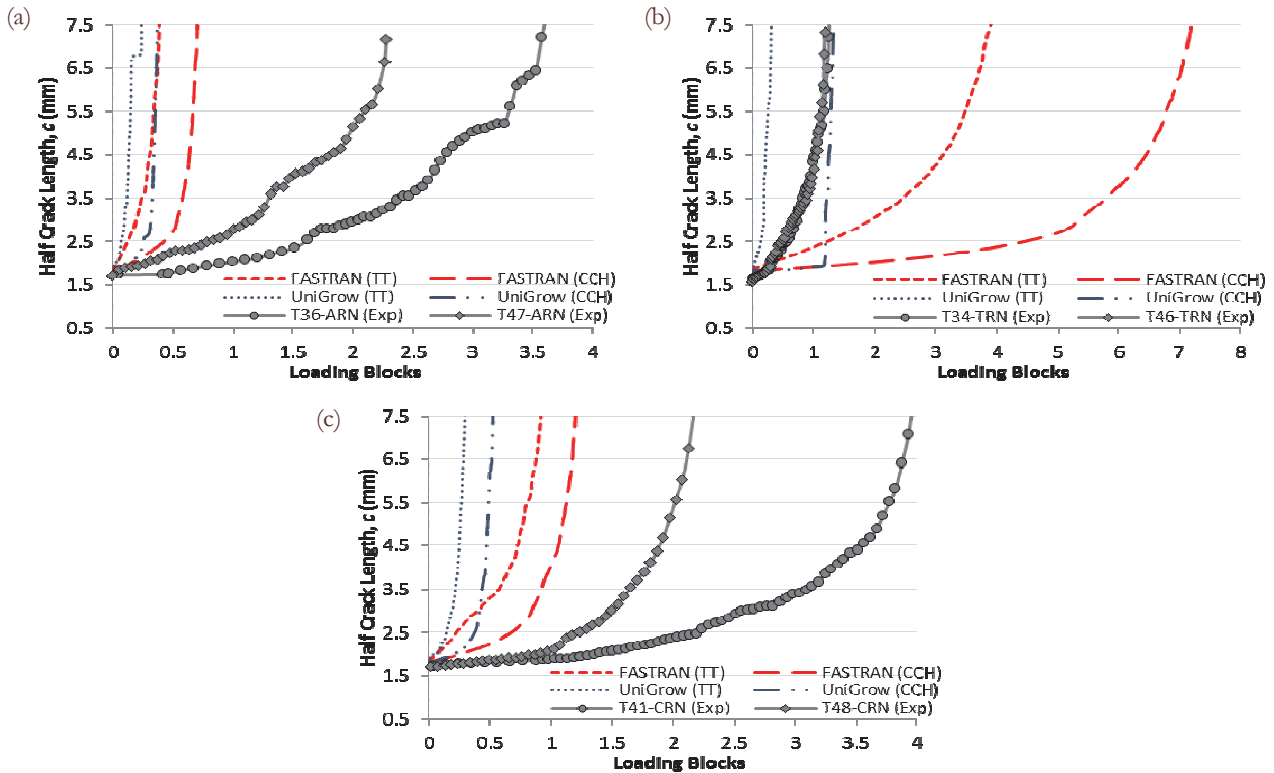


Figure 4: Variable amplitude crack growth curves for lowest loading level (a) axial, (b) torsion, and (c) combined axial-torsion loading.

Concerning loading history profile, the axial channel of the variable amplitude service loading history, as shown in Fig. 1(a), is composed of smaller cycles with significant tensile mean stress mixed with occasional larger cycles at a much smaller R ratio. The shear channel, on the other hand, is composed of small amplitude cycles, ranging between approximately zero to minimum conditions, mixed with occasional larger amplitude cycles of an approximate zero to maximum range. Given these differences, significantly more crack closure would be expected for mode I crack growth under pure torsion loading due to the smaller tensile mean stress values. This agrees with the more conservative FASTRAN life predictions for the axial loading history, as compared to the shear history. Residual stresses, on the other hand, are most significantly affected by the maximum stress values in a loading history. Therefore, the effect of the



loading history profile is not expected to have as large of an effect on residual stress distributions so long as the frequency and magnitude of maximum stress cycles are similar, which is the case in this study. As such, the fact that growth life predictions are more consistent between axial and torsion loading histories for UniGrow analyses, as compared to FASTRAN analyses, is not surprising.

In addition to load history profile, even for constant amplitude loading conditions, mode I crack growth rates for nominal torsion loading of the notched tubular specimens were considerably higher than those for axial nominal loading at the same applied SIF range. This was attributed to the presence of compressive tangential stress (T-stress) at the crack tip, resulting in an increased plastic zone size and crack driving force under multiaxial nominal stress states. Because UniGrow and FASTRAN are both meant to model crack growth under uniaxial nominal loading conditions, neither program accounts for multiaxial stress state effects, such as T-stress, on mode I crack growth rates. Therefore, for a given SIF range, both models will predict lower crack growth rates than what would be expected in experiments for torsion and combined axial-torsion nominal loadings. As a result, the overall reduction in conservatism for the pure torsion variable amplitude growth life predictions is to be expected.

Finally, crack growth analyses were performed for the variable amplitude combined axial-torsion (AT) loading histories. Given the tension dominated nature of the variable amplitude loading history applied in this study, crack growth behavior for the combined loading tests is expected to be similar to that observed for the axial only tests. By studying the results, this is found to be generally true. Figs. 3 and 4(c) show that crack growth life predictions for both axial only and combined loading tests tend to be conservative based on both FASTRAN and UniGrow analyses. Additionally, the degree of conservatism is found to increase with decreasing loading levels in both cases.

While crack growth prediction trends are qualitatively similar for the axial only and combined loading conditions, growth life predictions are generally found to be less conservative for combined loading tests. Similar to the growth life predictions for the pure torsion tests, some of this difference is likely due to the crack growth models' inability to account for increased growth rates due to the effect of T-stress on mode I crack growth. However, there are also additional factors that can contribute to this discrepancy which are only brought about under combined loading situations.

Fig. 1(b) shows that the variable amplitude service loading history investigated in this study contains a number of significant non-proportional loading events. When non-proportionally varying stresses are present in a crack growth analysis, it becomes especially difficult to calculate crack driving forces. Because the principal stress directions are not constant under non-proportional loading conditions, a growing crack is continuously subjected to varying degrees of mixed-mode loading, regardless of its orientation. Additionally, the tendency of a mode I crack to grow under the influence of maximum principal stress can cause increased crack meandering and crack face roughness as cracks try to align with the changing principal stress direction.

While there are many factors, in addition to plasticity induced closure and residual stress effects, which have the potential to influence crack growth behavior under variable amplitude combined loading conditions, some of these effects act to increase crack growth rates (e.g. T-stress and mixed-mode loading), while others tend to hinder crack growth (e.g. crack path meandering). For the loading conditions and specimen geometry of interest in the current study, the combined effect of all of these mechanisms appears to result in less conservative crack growth predictions for combined loading histories than for axial only histories. This agrees with the higher crack growth rates observed at higher SIF ranges for both in-phase and 90° out-of-phase constant amplitude combined loading conditions, when compared to those for axial loading.

## SUMMARY AND CONCLUSIONS

In general, variable amplitude crack growth predictions based on both FASTRAN and UniGrow analyses were found to be conservative, regardless of the initial crack geometry assumption, for both axial and combined axial-torsion loading conditions. The accuracy of crack growth predictions for pure torsion loading conditions, however, was found to vary depending on the crack growth model, although all predictions were less conservative than in the case of axial and combined loadings. This is despite the fact that, for both programs, the majority of experimental crack growth lives under constant amplitude axial loading conditions were predicted within a factor of  $\pm 3$  and generally found to fall between predictions based on the TT and CCH crack geometry assumptions. Additionally, comparisons with constant amplitude crack growth data show that the shift in conservatism between the different nominal loading conditions can likely be attributed to multiaxial stress state effects on mode I crack growth, such as the presence of T-stress and the potential for mixed-mode crack growth conditions. These effects are not accounted for in either crack growth model investigated.



## ACKNOWLEDGEMENTS

U.S. Naval Air Systems Command (NAVAIR) provided financial support for this study.

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