Effect of centrifugal load on crack path in thin-rimmed and webbed gears

F. Curà, A. Mura, C. Rosso
Politecnico di Torino, Italy
francesca.cura@polito.it, andrea.mura@polito.it, carlo.rosso@polito.it

ABSTRACT. Thin rimmed and webbed gears are used in particular applications to reduce systems weight. This kind of gears need an accurate and fail safe design. As a matter of fact, a possible failure, due to bending fatigue, consists in crack nucleation and consequent growth, in particular in the tooth root zone. These cracks may propagate through the tooth or through the rim. Crack propagation direction is basically influenced by the wheel geometry parameters, above all the rim thickness. Studies available in literature emphasize three ranges for the backup ratio values, involving different behaviors. These ranges are related to the crack propagation paths; respectively through the tooth, through the rim and in an unforeseeable way. This last uncertainty zone depends on other parameters, related to both geometry and loading conditions. In this work the effect of wheel speed related to the bending load has been investigated. The investigation has been carried out by means of numerical models involving both 2D finite element and extended finite element models (XFEM). Results shows that both crack initiation point and crack propagation path are strongly influenced by centrifugal load; this effect is mainly evident in the uncertainty zone of the backup ratio.

KEYWORDS. Fracture mechanics; Crack path; Crack propagation; Gears; XFEM

INTRODUCTION

Thin rim gears are used in those applications where weight reduction is needed, such as aerospace environment; in this field an accurate and fail safe design is required [1]. Fail safe design means that, if a failure occurs, serious consequences must be avoided. Considering thin rimmed gears, a possible failure, due to bending fatigue, consists in crack nucleation and consequent growth, in particular in the tooth root zone [2, 3]. These cracks may propagate through the tooth (causing a safe failure) or through the rim (causing a catastrophic failure). Crack propagation direction is basically influenced by the wheel geometry parameters, above all the rim thickness (ruled by the so called backup ratio). Studies available in literature emphasize three ranges for the backup ratio values, involving different behaviours [4, 5]. These ranges are related to the crack propagation paths, respectively through the tooth, through the rim and in an unforeseeable way. This last uncertainty zone depends on other parameters, related to both geometry and loading conditions, in particular considering as loading condition the centrifugal load (generated by the wheel rotation) that may significantly influence the crack growth path [6].

Concerning the effect of load conditions on crack propagation it is possible to find in literature some interesting works: Glodez et al. [7] experimentally investigated the effects of different load distributions along the tooth width. Pehan et al. [8] evaluated the effect of not uniform load distributions and the not uniformly crack growth along the tooth width, by

means of three dimensional simulations. Flasker et al. [9] evaluated the residual life of the wheel with a crack along the tooth root for different loading conditions considering the effect of the contact area on the crack propagation direction.
Only few works are available in literature about the effect of speed on thin rimmed gears: in particular Lewicki [6] investigated by experiment and 2D finite element models, the effect of the centrifugal load on crack propagation path; in this work a slotted spur gear is considered with notch machined along entire tooth face width. While Li, in its two works [10, 11] investigated effects of centrifugal load on bending strength, contact strength and deformations of a thin-rimmed spur gear used at high speed with the finite element method.
The aim of this work is to investigate the influence of centrifugal load on crack propagation path in thin rimmed and webbed gears.
According to literature, crack path is influenced by the initial crack position [12]. If it is assumed that initial crack is located at the maximum equivalent stress point, this point may shift its position according to both wheel rotation speed and external bending load.
In this work two effects of centrifugal load have been considered: the motion of the crack nucleation point and the change of crack propagation direction.
The first aspect has been investigated by means of finite element models where the centrifugal load has been varied and its influence on crack nucleation point has been observed.
The second analysis has been performed by means of extended finite elements models (XFEM) [13], where a crack has been growth in case of only bending stress and in the case of both bending and centrifugal load showing different behaviours.
Generally speaking, from the analysis of the obtained results it is possible to conclude that the wheel speed may cause the shifting of the point of maximum equivalent stress towards the tooth root, causing, in determinate cases, the failure mode changing (passing from safe failure, to catastrophic failure).
On the other hand if a nucleation point is fixed, the centrifugal load tends to growth the crack in radial direction by turning down the crack propagation direction.

NUMERICAL MODELS

First of all a numerical investigation about the effect of centrifugal load on crack initiation point has been carried on. Assuming that the crack may nucleate at the point where the maximum equivalent stress is achieved, the effect of speed (with respect to the bending load) has been investigated by means of 2D FE models developed by MSC.Partan/Nastran® software.

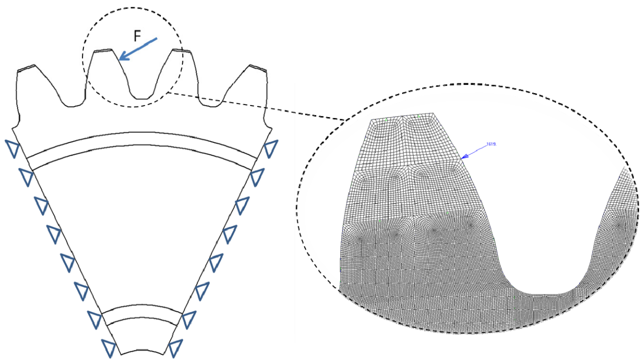

Figure 1: 2D finite elements model and mesh refinement.

A steel gear with the following geometrical parameters has been modelled: modulus 7.4mm, 28 teeth, 30° pressure angle, 8mm face width, as in [1, 2].
Concerning the 2D model, in order to reduce the computation time, the gear has been cut and only a sector of four teeth has been considered. The geometry of two teeth has been subdivided in small sectors in order to obtain little portions with regular shapes to allow a more regular and an easier meshing process (see Fig. 1).
The average element edge in the root region where the maximum stresses are located is 0.1 mm. The mesh consists in "quad" elements with four nodes. Boundary conditions consist in displacement loads on the lateral edges of the sector to

simulate the presence of the whole gear. The gear is loaded with a force on the highest point of single tooth contact (HPSTC).
Tab. 1 resumes the test cases considered for the 2D FEM analysis.

Test Case	Bending Force [N]	Speed [rpm]
1	1619	0
2	0	10000
3	1619	10000
4	3238	10000
5	8095	10000
6	12952	10000

Table 1: Test cases considered for the first 2D FEM analysis (the wheel is a full gear).

Crack propagation studies have been performed by means of 3D extended finite element models (XFEM) [12]. In this case the entire gear geometry has been considered. XFEM region has been created by defining three zones near the tooth root where the crack has been hypothesized to propagate; the average elements size in each region is respectively: 1.6mm, 0.6mm and 0.3mm.
An elliptical crack has been inserted at one extremity of the tooth width (with principal axis dimensions respectively: 0.1mm and 0.25mm), see Fig. 2.
Each simulation consists in 15 propagation steps, with an increment of crack length equal to 0.3mm at step; the Paris' law has been used as propagation law.
The load has been placed as a force distributed along a line on the tooth width at the HPSTC, in combination with centrifugal load. The hub diameter has been clamped.

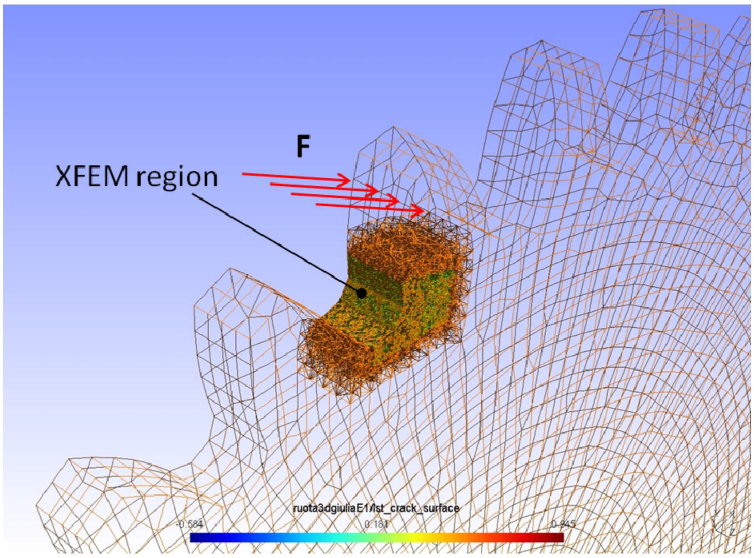

Figure 2: 3D extended finite element model (XFEM).

Propagations have been calculated in gears with different rim thicknesses and web thicknesses (respectively defined by the backup ratio m_b = B/m and the web ratio m_w = W /L, where B is the rim thickness, m is the gear module, W is the web thickness and L is the face width, see Fig. 3) [12, 14], in order to evaluate if these geometric parameters may affect the contribution of the centrifugal load on crack propagation direction.
Fig. 3 shows the meaning of the geometric parameters backup ratio (m_b) and web ratio (m_w). A so called full gear is a wheel whose web thickness is equal to the face width (m_w = 1).
Tab. 2 resumes the XFEM simulation run in this work.

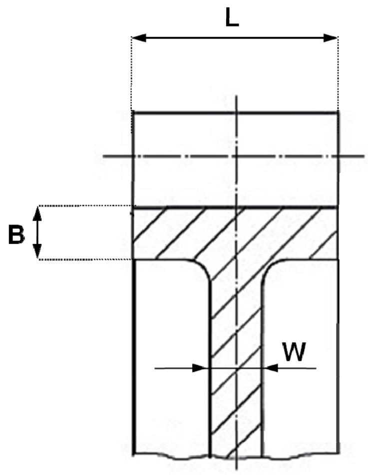

Figure 3: Geometric parameters of the wheel.

Test Case	Backup ratio m_b	Web ratio m_w	Load
1	Full gear		Bending
2	2.16	0.125	Bending
3	1.08	0.012	Bending
4	0.87	0.125	Bending
5	Full gear		Bending + speed
6	2.16	0.125	Bending + speed
7	1.08	0.012	Bending + speed
8	0.87	0.125	Bending + speed

Table 2: Test cases considered for the crack propagation study with XFEM.

RESULTS AND DISCUSSION

Static analysis

Firstly, the interaction between bending force, centrifugal field and position of the maximum stressed point has been investigated. The model considered in this case is a solid gear without any thin rim or web. For doing that, a first analysis was carried on with the following parameters:

1. the centrifugal field has been set as constant (rotation speed = 10000 rpm), the value of the force has been increased and the crack direction investigated;
2. the force has been always applied at HPSTC and its magnitude has been changed in order to highlight the effect of the interaction between the two loads.

In particular four models have been developed, starting from a value of the bending force F and then its magnitude has been increased at respectively 2F, 5F and 8F.

Fig. 4 shows the results obtained from this study; it is possible to observe that, if the centrifugal load is dominant, the crack tends to propagate approximately on a radial direction, while, if the bending effect is higher then the centrifugal field, the crack tends to propagate through the tooth.

The interaction between the bending force and the centrifugal field produced by the rotational speed on the position of the maximum equivalent stress (Von Mises) point has been also investigated.

The aim of this second analysis is to study the combinations of the two loads that produces the maximum allowable stress (in this case the material yield stress is considered as maximum stress) in order to verify how the position of the maximum allowable stress may shift by changing these two parameters. The yield stress of the material considered in this work is 1034 MPa.

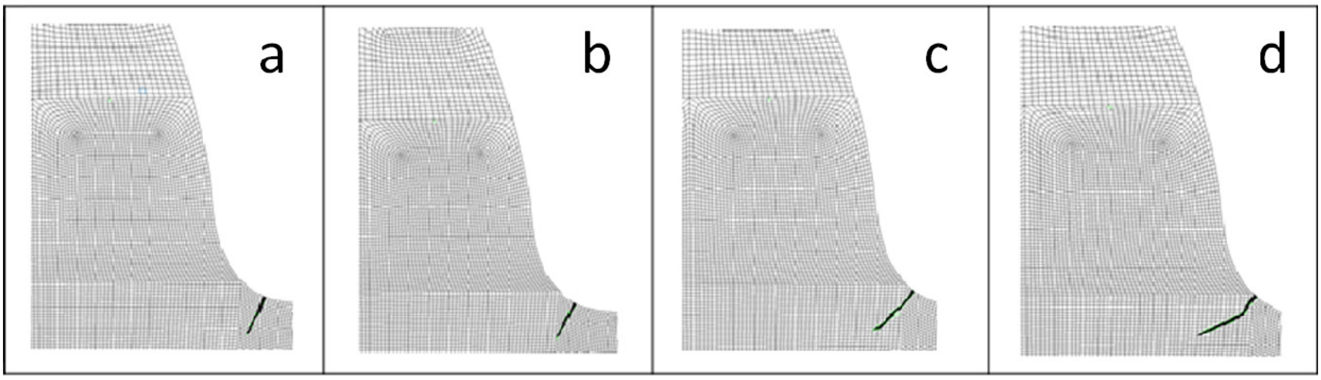

Figure 4: Interaction between bending and centrifugal load on the crack propagation direction: a) F + rotation; b) 2F + rotation; c) 5F + rotation; d) 8F + rotation.

Firstly, the value of the bending force (applied at the highest point of the single tooth contact HPSTC) that alone (without centrifugal load), may produce yield stress has been obtained and the relative element, where the stress has been reached, has been identified. Consequently, the speed value that, alone (without bending force applied), produces the yield stress, and the corresponding element has been determined.
Fig. 5 shows the two elements where the yield stress has been reached: the green one is the most stressed one subjected only to the bending force and the red one is the most stressed with only the centrifugal field. In the model considered in this work, between these two limit elements there are 15 elements, so a range of about 1.5 mm, on a root fillet with radius equal to 3mm, represents the 33% of the fillet length.

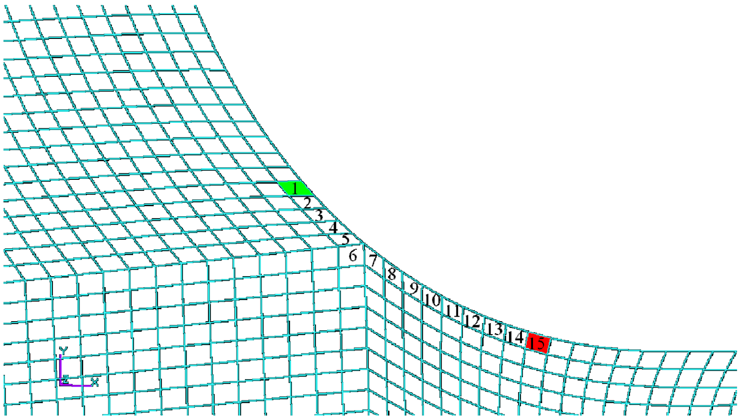

Figure 5: Elements where the yield stress is reached (green: only bending force applied, red: only centrifugal load applied).

Any combination of these two loads gives the maximum stress element between these extremes, the tensors of tensions has been evaluated in all these elements for both these limit conditions.
For each element between these two extremities elements, two data have been collected: σ_b (stress tensor evaluated in condition of maximum force) and σ_c (stress tensor evaluated in condition of maximum speed). Once this result has been reached, other different combinations have been considered.
To take into account all possible combinations of these two loads, a Matlab script has been employed. Considering the combination of these stresses as linear, the implemented formula is:

$$A \cdot \sigma_b + B \cdot \sigma_c = \sigma_{yeld} \quad (1)$$

Eq. 1 is characterized by the presence of two coefficients, *A* and *B,* representing the contribution factors for the two stress tensors to the yield condition. In other words, when the only bending is present and creates yield condition, *A* is equal to one and *B* is equal to zero.
The contrary occurs when only the centrifugal force is present. So a condition where both loads are active is represented by a combination of the two limit stress tensors and *A* and *B* represent the ratio to the limit conditions. Using that reasoning it is possible to study the interaction between the two loading conditions and all possible combinations can be easily studied.
For analysing all possible conditions, the Von Mises formula has been applied on the computed stress tensors and solved with respect to *A* and *B*, by satisfying the following condition:

$$\sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau^2} - \sigma_{yeld} < tol \qquad (2)$$

where *tol* is a tolerance value fixed on 10 MPa.
The problem is obviously under-constrained, but solving it for all the possible combinations of *A* and *B*, the variation of maximum stress point can be highlighted. As a matter of fact, every time that the condition of eq. 2 is verified, it means that this load combination produced yield stress, so this couple of values is critical and the most stressed element may be detected. Fig. 6 shows all possible load combinations that have been determined.

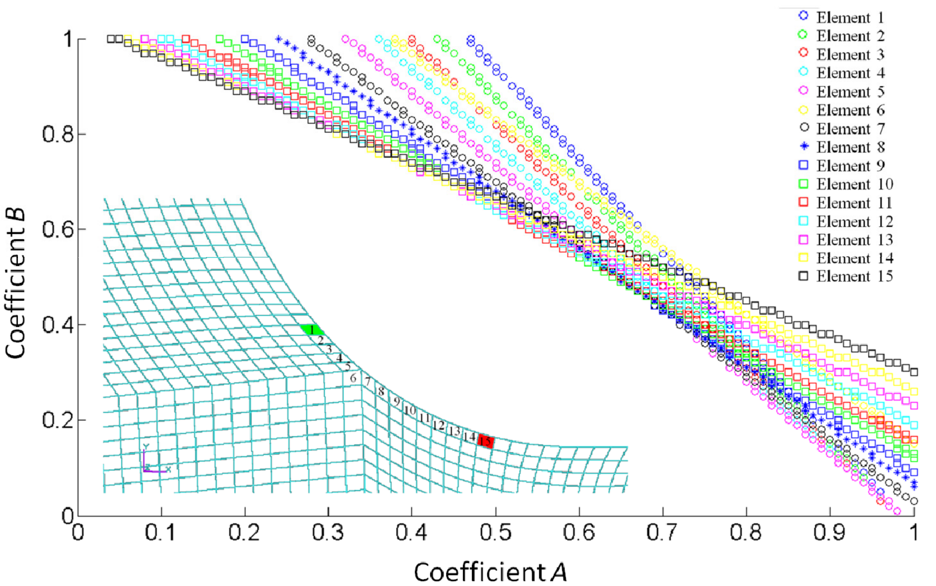

Figure 6: Combination of *A* and *B* and the relative most stresses element.

From Fig. 6 it is possible to observe that, for a defined *A* value, there are many *B* values that may produce critical stresses and so the most stressed element is different.
Fig. 7 shows the limit curve where a defined *A* value corresponds to the first *B* value that produces yield stress and the corresponding element is highlighted. The curve of Fig. 7 is the envelope of all the possible conditions that produce yield in the tooth, so it could be used to understand the mutual effect of the two loading conditions and the possible location of the maximum stress.
In that way, it is possible to relate the loading conditions to the crack nucleation point and hence give a forecast of the crack propagation behavior. Combining the geometrical features of the gear (backup and web ratii) with the curve of Fig. 7, it is possible to have a design indication for preventing catastrophic failures.

Propagation Analysis

Once highlighted the relationship between variation of loads and maximum stress points, the influence of centrifugal force with respect to the geometry has also been studied. In particular, the interaction between centrifugal load and geometrical parameters has been investigated considering the propagation effect.

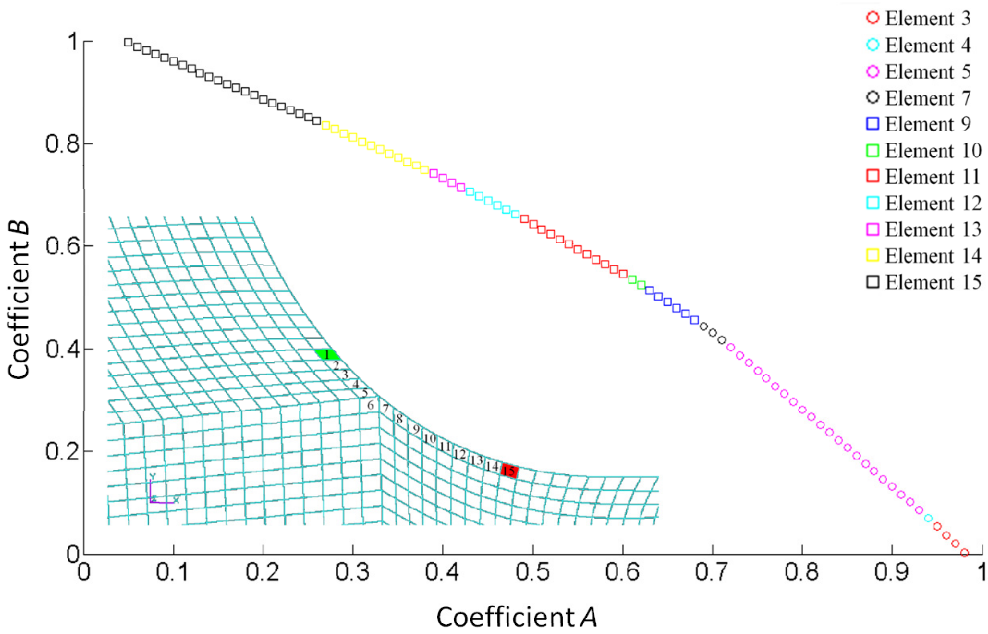

Figure 7: Limit curve that highlights elements that reaches the yield condition before others.

For doing that, a XFEM analysis has been carried on. Propagation analysis has been performed in four cases by varying rim and web thicknesses, as shown in Tab. 2. In all cases the crack initiation has been placed at the point where the maximum equivalent stress is reached by applying only the bending force, then the propagation simulations have been run for each test case without and with the centrifugal load, in order to evaluate how the speed affects the propagation direction. This is for highlighted the only effect of the rotational speed and geometrical features.

Figs. 8 to 11 show the obtained results comparing the cracks propagated with and without centrifugal load (the bending force is always present). In particular in Fig. 8 is shown the case of full gear, in Fig. 9 a wheel with backup ratio $m_b = 2.16$ and web ratio $m_w = 0.125$, In Fig. 10 a gear with $m_b = 1.08$ and $m_w = 0.012$ and in Fig. 11 a gear with $m_b = 0.87$ and $m_w = 0.125$.

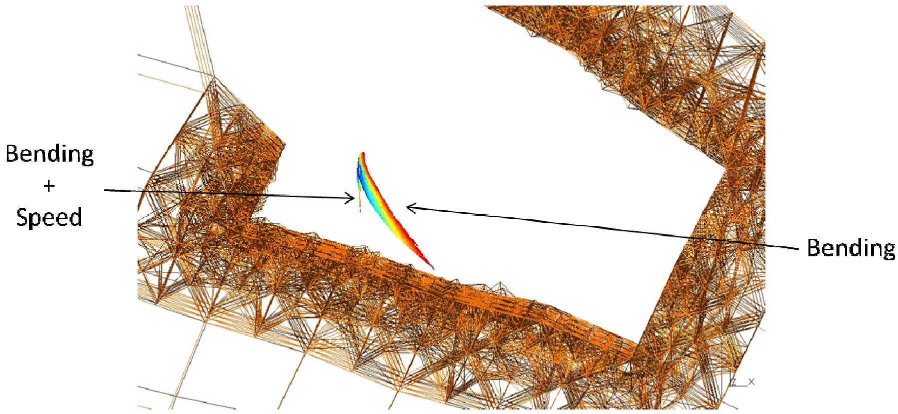

Figure 8: Propagation in full gear.

Generally speaking, it is possible to observe that, in simulations with the centrifugal load, the crack path follows a direction that approaches the radial direction, independently by the geometrical features.

As demonstrated in literature, the crack propagation related to the only bending action is ruled by the gear geometry; considering the effect of the centrifugal force, geometry gets less important and rotational speed always shifts the crack propagation towards a catastrophic failure.

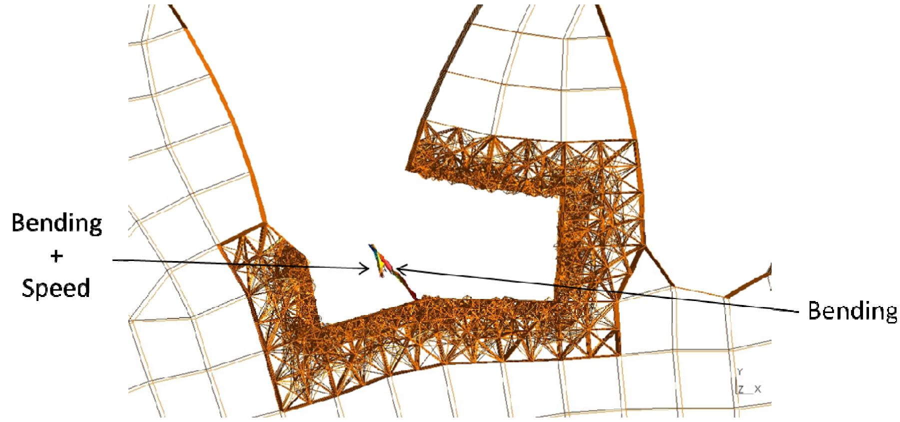

Figure 9: Propagation in m_b = 2.16, m_w =0.1205.

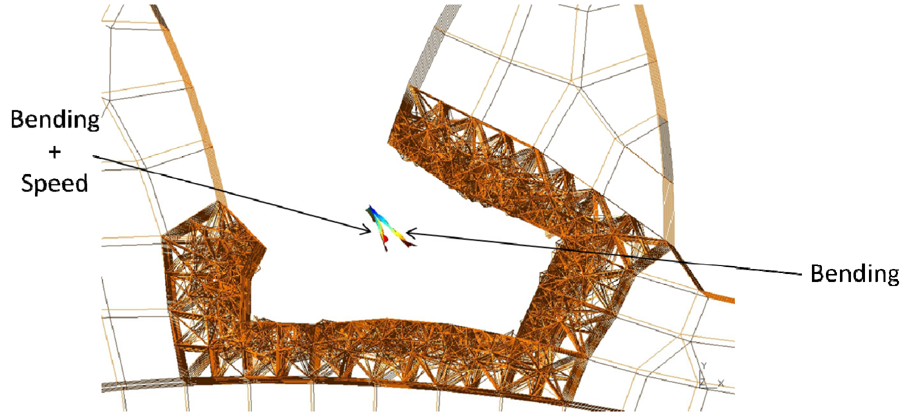

Figure 10: Propagation in m_b = 1.08, m_w =0.012.

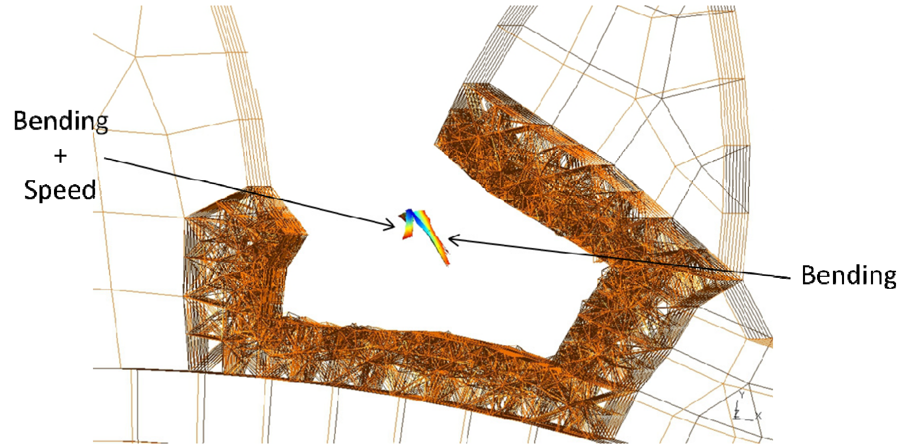

Figure 11: Propagation in m_b = 0.87, m_w =0.1205.

CONCLUSIONS

In this work the effect of the speed and in particular the interaction between the bending force and the centrifugal load on crack propagation path in thin-rimmed and webbed gears has been investigated.
Firstly a static 2D finite elements study has been carried on to investigate how the above quoted loads may influence

the position of the point on the tooth root where the maximum equivalent stress has been reached (this point is assumed as the crack initiation).

Results show that the centrifugal load tends to shift the crack initiation point at the bottom of the tooth root fillet while, if bending load is predominant the nucleation point may shift to the top edge of tooth root fillet. An equation has been introduced to correlate the maximum equivalent stress position to the amount of bending and centrifugal loads.

Then a propagation study has been carried on by means of extended finite elements models. The aim of these simulations has been to investigate how the centrifugal load may affect the crack path direction: cracks, initiated at the same point, have been propagated with and without centrifugal load, showing that, in presence of non negligible centrifugal loads, cracks tends to propagate in radial direction.

From works available in the literature it is possible to state that in thin rim gears crack propagation direction mainly depends on gear geometry (rim and web thickness) and crack initiation point.

From the analysis of the results presented in this work it is possible to conclude that the centrifugal load strongly influences both crack initiation point and crack propagation direction, by shifting the crack initiation point at the bottom of the tooth root fillet and by driving the crack propagation in the radial direction.

Generally speaking centrifugal loads may be the key factor to promote failsafe or catastrophic failures in all those cases where the crack propagation path is not only defined by the gear geometry (uncertainty zone), causing the centrifugal load an important effect in both crack initiation position and crack propagation direction.

REFERENCES

[1] Lewicki David G., Crack Propagation Studies to Determine Benign or Catastrophic Failure Modes for Aerospace Thin-Rim Gears, NASA Tecnical Memorandum 107170.

[2] Lewicki David G., Three-Dimensional Gear Crack Propagation Studies, U.S. Army Research Laboratory, Lewis Research Center, Cleveland, Ohio, NASA/TM-1998-208827.

[3] Lewicki David G., Ballarini R., Rim thickness effects on gear crack propagation life, International Journal of Fracture, 87 (1997) 59-86.

[4] Lewicki David G., Gear Crack Propagation Path Studies, Guidelines for Ultra-Safe Design- NASA/TM-2001-211073.

[5] Kramberger J., Flasker J. – Numerical Simulation of 3-D Crack Growth in Thin-Rim Gears – University of Maribor, Faculty of Mechanical Engineering, Smetanova 17, SI-2000 Maribor, Slovenia.

[6] Lewicki David G., Effect of Speed (Centrifugal Load) on Gear Crack Propagation Direction, U.S. Army Research Laboratory, Glenn Research Center, Cleveland, Ohio (2001).

[7] Glodez S., Pehan S., Flasker J., Experimental results of the fatigue crack growth in a gear tooth root, Int. J. Fatigue, 20 (1998) 669-675.

[8] Pehan S., Hellen Trevor K., Flasker J., Glodez S. – Numerical methods foe determing stress intensity factors vs crack depth in gear tooth roots, Int. J. fatigue, 19 (1997) 677-685.

[9] Flasker, J., Glodez, S., Pehan., S., Influence of contact area on service life of gears with crack in tooth root, Communications in Numerical Methods in Engineering, 11 (1995) 49-58.

[10] Li, S., Centrifugal load and its effects on bending strength and contact strength of a high speed thin-walled spur gear with offset web, Mechanism and Machine Theory, 43 (2008) 217–239.

[11] Li, S., Effects of centrifugal load on tooth contact stresses and bending stresses of thin-rimmed spur gears with inclined webs, Mechanism and Machine Theory, 59 (2013) 34–47.

[12] Curà, F., Mura, A., Rosso, C., Investigation about crack propagation paths in thin rim gears, Fracture and Structural Integrity, 30 (2014) 446-453. DOI: 10.3221/IGF-ESIS.30.54.

[13] Sukumar, N., Moës, N., Moran, B., Belytschko, T., Extended finite element method for three-dimensional crack modelling, International Journal for Numerical Methods in Engineering, 48(11) (2000) 1549–1570.

[14] Curà, F., Mura, A., Rosso, C., Effect of rim and web interaction on crack propagation paths in gears by means of XFEM technique, Fatigue Fract. Eng. Mater. Struct. (2015) (article in press).