Understanding the edge crack phenomenon in ceramic laminates

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ABSTRACT. Layered ceramic materials (also referred to as "ceramic laminates") are becoming one of the most promising areas of materials technology aiming to improve the brittle behavior of bulk ceramics. The utilization of tailored compressive residual stresses acting as physical barriers to crack propagation has already succeeded in many ceramic systems. Relatively thick compressive layers located below the surface have proven very effective to enhance the fracture resistance and provide a minimum strength for the material. However, internal compressive stresses result in out-of plane stresses at the free surfaces, what can cause cracking of the compressive layer, forming the so-called *edge cracks*. Experimental observations have shown that edge cracking may be associated with the magnitude of the compressive stresses and with the thickness of the compressive layer. However, an understanding of the parameters related to the onset and extension of such edge cracks in the compressive layers is still lacking. In this work, a 2D parametric finite element model has been developed to predict the onset and propagation of an edge crack in ceramic laminates using a coupled stress-energy criterion. This approach states that a crack is originated when both stress and energy criteria are fulfilled simultaneously. Several designs with different residual stresses and a given thickness in the compressive layers have been computed. The results predict the existence of a lower bound, below no edge crack will be observed, and an upper bound, beyond which the onset of an edge crack would lead to the complete fracture of the layer.

KEYWORDS. Edge crack; Residual stresses; Ceramic laminate; FE analysis; Coupled criterion.

INTRODUCTION

Ceramics have been used for many decades as structural elements, but due to their inherent brittleness they have been mainly utilized under compressive loading conditions. Nowadays, most of the new engineering designs need to withstand tensile stresses which imply potential limitations for ceramics due to their low fracture toughness and the sensitivity of ceramic material strength to the presence of defects [1-3]. The brittle fracture of glasses and ceramics is a consequence of the material defects located either within the bulk or at the surface, resulting from the processing and/or machining procedures [4, 5]. Under external applied stress, the stress concentration associated with such defects is the common source of failure of ceramic components. If each defect is considered as a crack or a potential source for crack initiation, then obviously the size and type of these defects determine the mechanical strength of the material [6].

The distribution of defects of different sizes within a ceramic component yields a statistically variable strength which can be described by the Weibull theory [7, 8]. As a consequence of such behaviour there remains a (small) probability of failure even at very small applied loads (*i.e.*, no lower threshold for strength). Since flaws are intrinsic to processing and in most cases unavoidable, the mechanical reliability of ceramic components is associated with such a flaw distribution. In order to reduce both the defect population and the defect size, many studies have been devoted in the past to improve ceramic processing [9]. Another approach to increase strength has been to introduce compressive residual stresses at the surface. A successful example can be found in strengthened glass [10] and more recently Gorilla glass [11]. However, a significant reduction of strength variability cannot be achieved with these approaches. In an attempt to reduce the level of uncertainty in mechanical strength and to overcome the lack of toughness of structural and functional ceramics, newer approaches have been developed in which knowledge about the energy release mechanisms has resulted in the creation of "flaw tolerant" (strength reliable) materials, with improved fracture toughness [9, 10, 12-27].

Layered ceramic materials (also referred to as "ceramic laminates") are becoming one of the most promising areas of materials technology. They have been proposed as an alternative for the design of structural ceramics with improved fracture toughness, strength and mechanical reliability. Among all, laminates designed with strong interfaces and compressive residual stresses have led to an increase in fracture energy, thermal shock resistance and, in some cases, a decrease in the sensitivity of the material strength to the different size of defects. The utilization of tailored compressive residual stresses acting as physical barriers to crack propagation has succeeded in many ceramic systems [17, 23, 25, 28-31].

However, a limiting factor in the design of these multilayer systems is the fact that the beneficial compressive stresses in one type of layers have to be balanced by (potentially) critical tensile stresses in the other layers. Therefore, the use of relatively high residual stresses to enhance the mechanical behaviour can lead to the onset of initial cracks in the layers, which may later propagate in service under external applied stresses, leading to failure of the component. Fig. 1 illustrates typical surface cracks associated with residual stresses in planar ceramic-ceramic multilayer systems. An example is that of "tunnelling cracks", which may appear at the free surface of the layers with tensile stresses and are oriented perpendicular to the layer plane [32, 33]. Another type of cracks are the so-called "edge cracks", which initiate from pre-existing flaws at the free surface of compressive layers, oriented parallel to the layer plane [34, 35]. The third type is delamination, mainly occurring at the corner interface between adjacent layers.

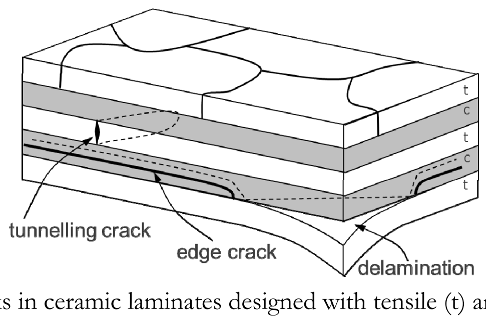

Figure 1. Schematic of surface cracks in ceramic laminates designed with tensile (t) and compressive (c) residual stresses.

Experimental observations have shown that edge cracking may be associated not only with the magnitude of internal compressive stresses but also with the thickness of the compressive layer [26]. Some authors have speculated that this phenomenon may be related to crack bifurcation observed in some laminates with relatively high compressive residual stresses [36, 37]. In a recent work, the authors applied a stress-energy criterion to set the conditions for edge cracking formation and extension in a particular laminate [38]. However, an understanding of the parameters related to the onset and extension of such edge cracks in the compressive layers is still lacking. In this work, a 2D parametric finite element model has been developed to predict the onset and propagation of an edge crack in ceramic laminates. The FE model utilizes the coupled stress-energy criterion [39], which considers simultaneously the necessary stress and energy conditions for the onset of the edge/tunnelling cracks and their further propagation. Several cases have been computed to show the effect of the compressive residual stresses on the onset and extension of edge cracks in ceramic laminates.

EXPERIMENTAL OBSERVATIONS

The ceramic laminate of study consists of 9 alternated layers combining two ceramic materials: (i) alumina with 5% tetragonal zirconia, named as ATZ, (ii) alumina with 30% monoclinic zirconia, referred to as AMZ. Fig. 2 shows a schematic of a prismatic bending bar with approx. dimensions of (LxBxH) 45mm x 4mm x 3mm. Due to the different thermal strains during cooling down from the sintering temperature, elastic strains are generated in the ATZ and AMZ layers, which lead to internal (in-plane) residual stresses. In this particular case, the ATZ layers have tensile stresses and the AMZ layers compressive stresses. For more details on the estimation of residual stresses see [26, 35].
In Fig. 2 an edge crack along the centre of the AMZ layer can be clearly seen. This is due to the tensile stress component generated at the free surface, having its maximum value at the edge, and decreasing into the material (see [34] for more details).

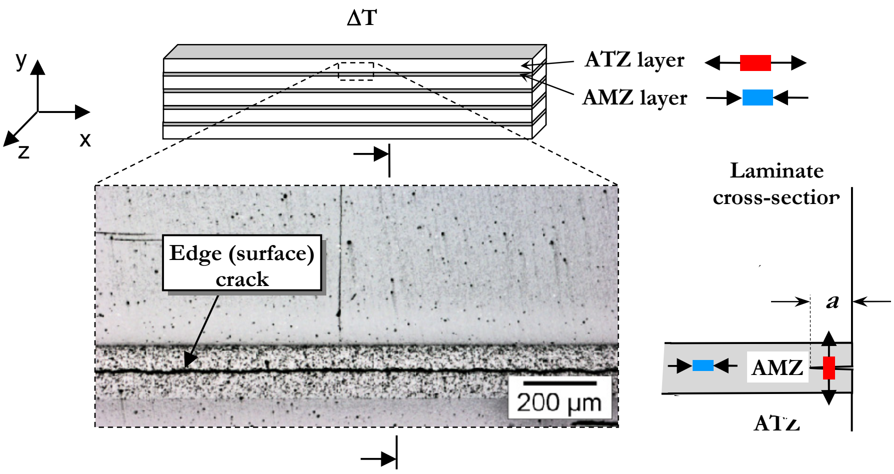

Figure 2. Experimental observation of the edge crack phenomenon in compressive AMZ layer, and stress redistribution at the free surface of the thinner compressive layer.

NUMERICAL ANALYSIS OF EDGE CRACKING IN LAMINATES

FE Model

The FE model prepared for the studies has been designed as a fully parametric model which enables automatic creation of any arbitrary laminate configuration with different volume ratios of particular material components (leading to different levels of residual stresses) or different thicknesses of the AMZ layer. Based on the real specimens, the considered model consists of nine layers composed by alternating ATZ and AMZ materials. The height and width of the laminate was fixed for all simulations to be H=3mm and W=4mm, respectively. The thickness of the inner AMZ layer was varied in the interval (30-350μm) and the thicknesses of the ATZ layers correspondingly tailored to reach the total thickness of 3mm. The material properties for the ATZ and AMZ layers needed for the numerical simulation are listed in Tab. 1.
To simulate (i) the nucleation and (ii) the propagation of the edge crack and its dependency on the layer thickness and level of residual stress inside the layer, a 2D FE model of the laminate cross-section was created – see Fig. 3. Quadratic PLANE183 elements with generalized plane strain option were used to correctly capture the thermoelastic stress state in the laminate. FE software ANSYS 15.0 was used for this purpose. An example of the FE mesh is shown in Fig. 3. The

thickness of the inner AMZ layer is set to values between 30μm and 350μm; the length (depth) of the edge crack into the AMZ layer can be set in the interval 0μm to 400μm. For every thickness of the AMZ layer, a large number of simulations with different edge crack lengths was done. In the first step, when the edge crack length is equal to 0μm (no edge crack) the stresses σ_{yy} along the prospective crack path and potential energy of the uncracked body are calculated and saved for further postprocessing. In the next simulations, the edge crack of a given length is introduced in the model and the actual potential energy of the cracked body (for each crack length) is calculated again and used later for calculation of the incremental energy release rate $G_{inc}(a)$ and the energy release rate (ERR) $G(a)$ at the crack tip as follows:

$$\begin{aligned} G_{inc}(a) &= -\left(W(0)-W(a)\right)/a \\ G(a) &= -dW(a)/da \end{aligned}, \qquad (1)$$

where W is the potential energy which depends on the crack length – see [38].
For verification purposes, the ERR G(a) at the crack tip was also calculated in ANSYS simulation using the implemented CINT function (employing J-integral). It was found that it leads to practically same curves of ERRs as the energy approach based on Eq. (1). The biggest advantage of the energy approach is that we can directly calculate the incremental ERR $G_{inc}(a)$ as well as the ERR at the crack tip $G(a)$. If the contour integral method is to be used, more care about the used mesh is necessary and also a recalculation of $G(a)$ to $G_{inc}(a)$ has to be made in the postprocessing.

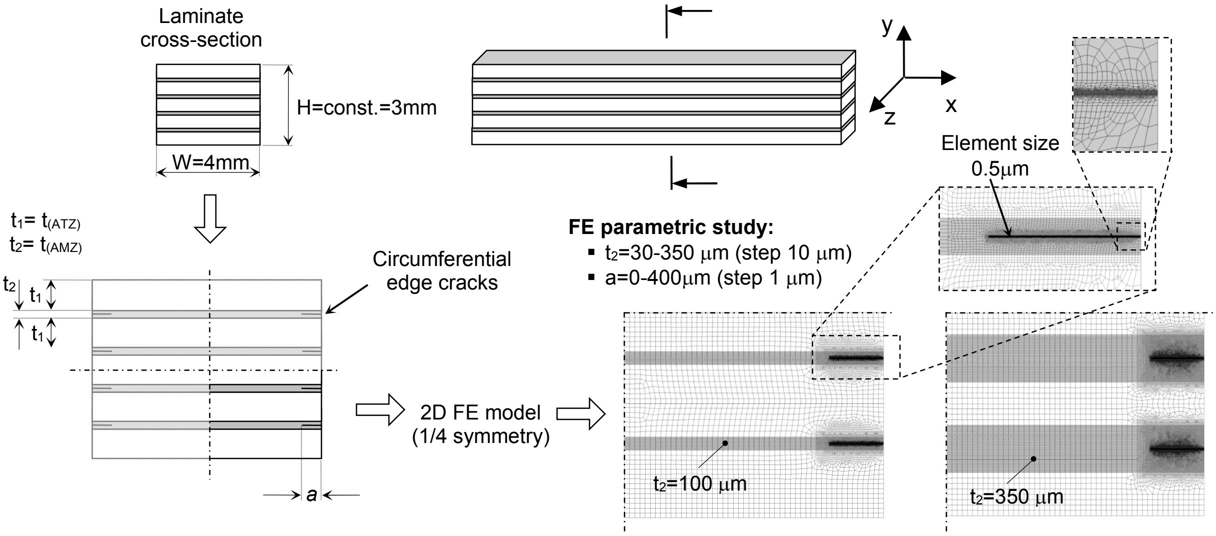

Figure 3. Schematic of the fully parametric FE model, developed to study the propagation of edge cracks in the compressive AMZ layers.

Material	E [GPa]	ν [-]	α x10[6] [K^{-1}]	σ_c [MPa]	K_{Ic} [MPa.m$^{1/2}$]	G_c [J/m^2]
ATZ	390±10	0.22	9.8±0.2	422±30	3.2±0.1	25±2
AMZ	280±10	0.22	8±0.2	90±20	2.6±0.1	23±2

Table 1: Material properties of the ATZ and AMZ layers [40].

Coupled stress-energy criterion
In order to determine the suitable conditions for the onset of the edge crack, the coupled stress-energy criterion was employed – see [39, 41]. This criterion states that a crack originates if two conditions (i.e. stress and energy conditions) are fulfilled simultaneously – namely $G_{inc}(a) \geq G_c^{(AMZ)}$ and $\sigma_{yy} \geq \sigma_c$. The first condition means that there is enough energy

available to create a crack, and the second condition stipulates that the tensile stress is greater than the tensile strength all along the new presumed crack path.

To predict the onset of such an edge crack, the coupled criterion [39, 41] allows getting rid of any assumption on the existence of flaws able to trigger cracking [36, 37]. There is no adjustable parameter in the coupled criterion, whereas in the other case the flaw size is selected so as to fit with the experimental measures. This choice is somewhat arbitrary because it does not rely on micrographic observations. Moreover the flaw approach analyses the early stage of the edge cracking and assumes a further crack growth in depth and along the specimen faces (channelling) to reach the observable state. On the other hand, with the coupled criterion we make the assumption that the crack appears almost simultaneously all around the specimen and then grows in depth.

Application to ceramic laminates

For demonstration, let us consider our laminate of study, where the thickness of the AMZ layer is selected as 0.150mm. After the sintering process the laminate is cooled down from the reference (sintering) temperature - lying between 1200-1500°C - to room temperature which results in origination of compressive residual stresses inside the AMZ layer (and tensile stresses in ATZ layer). In case that the difference to the reference temperature is relatively low (e.g. ≈ 700°C), the magnitude of compressive residual stresses will be relatively low (in this particular case ≈-400MPa) and no edge crack can be initiated, because at no point both energy and stress criterion is fulfilled ($\sigma_{yy} \geq \sigma_c$ and $G_{inc}(a) \geq G_c^{(AMZ)}$) – see Fig. 4a). If the specimen is further cooled down until a value of residual stress $\sigma_{res}^{(AMZ)}$=-432MPa inside the AMZ layer, then a first point of satisfaction of both criteria is reached (namely $G_{inc}(a) = G_c^{(AMZ)}$) and the edge crack is originated with a sudden jump from zero length to length a_1=0.059mm as depicted in the Fig. 4b). After continuation of the cooling down process, the residual stresses inside the AMZ layer become yet lower and can theoretically reach values around -700MPa. This state is demonstrated in the Fig. 4c). Once the crack is nucleated, then it propagates through the AMZ layer as long as the ERR at the crack tip $G(a)$ is greater than $G_c^{(AMZ)}$. The final crack length after the cooling down process is thus determined by point 3 in the Fig. 4c) and reaches a value of a_3. If the curve of $G(a)$ become higher than $G_c^{(AMZ)}$ at all points along the possible edge crack path, then the AMZ layer should break totally in the whole plane of the specimen. This can thus indicate not suitable laminate configurations for which the processing of the specimens would become problematic.

RESULTS

With the introduced model a parametric study involving an influence of the AMZ layer thickness and level of residual stress inside this layer on the origination and propagation of the edge crack has been carried out. The aim was to understand how the mentioned parameters influence the edge cracking phenomenon. A large number of different laminate configurations were calculated in order to obtain general insight into the problem.

Let us consider now several laminate configurations with different thicknesses of the AMZ layer, ranging from 50 to 350μm. If the total height of the laminate is kept constant, then these configurations will result in different volume ratios of ATZ and AMZ components and thus also in different levels of residual stresses (upon the assumption of a complete cooling down process from 1300°C to room temperature). In order to enable a comparative study for a given level of residual stress, a corresponding ΔT has to be first calculated for each laminate configuration. Then a further set of simulations is performed by introducing an edge crack with a length between 0μm and 400μm (with a step of 1μm), enabling the estimation of G, G_{inc} and σ_{yy} (along the prospective crack path) as a function of the edge crack length a.

Consider now a level of residual stress in the AMZ layer to be $\sigma_{res}^{(AMZ)}$=-200MPa. The dimensionless ERR and tangential stress σ_{yy} ahead the crack tip are plotted together in Fig. 5a. This plot clearly shows that no edge cracking is possible. If the level of compressive stress is increased to the value $\sigma_{res}^{(AMZ)}$=-300MPa as shown in Fig. 5b, we can clearly see that for AMZ layers thicker or equal to 285μm edge cracking can be predicted, since both conditions $\sigma_{yy} \geq \sigma_c$ and $G_{inc}(a) \geq G_c^{(AMZ)}$ are fulfilled simultaneously to certain length of the edge crack indicated by point 1 in the referred graph. Once the edge crack is created, it propagates until ERR at the crack tip is higher than the fracture toughness of the material $G(a) \geq G_c^{(AMZ)}$. Again for layer thicknesses lower than 285μm no edge cracking is to be expected.

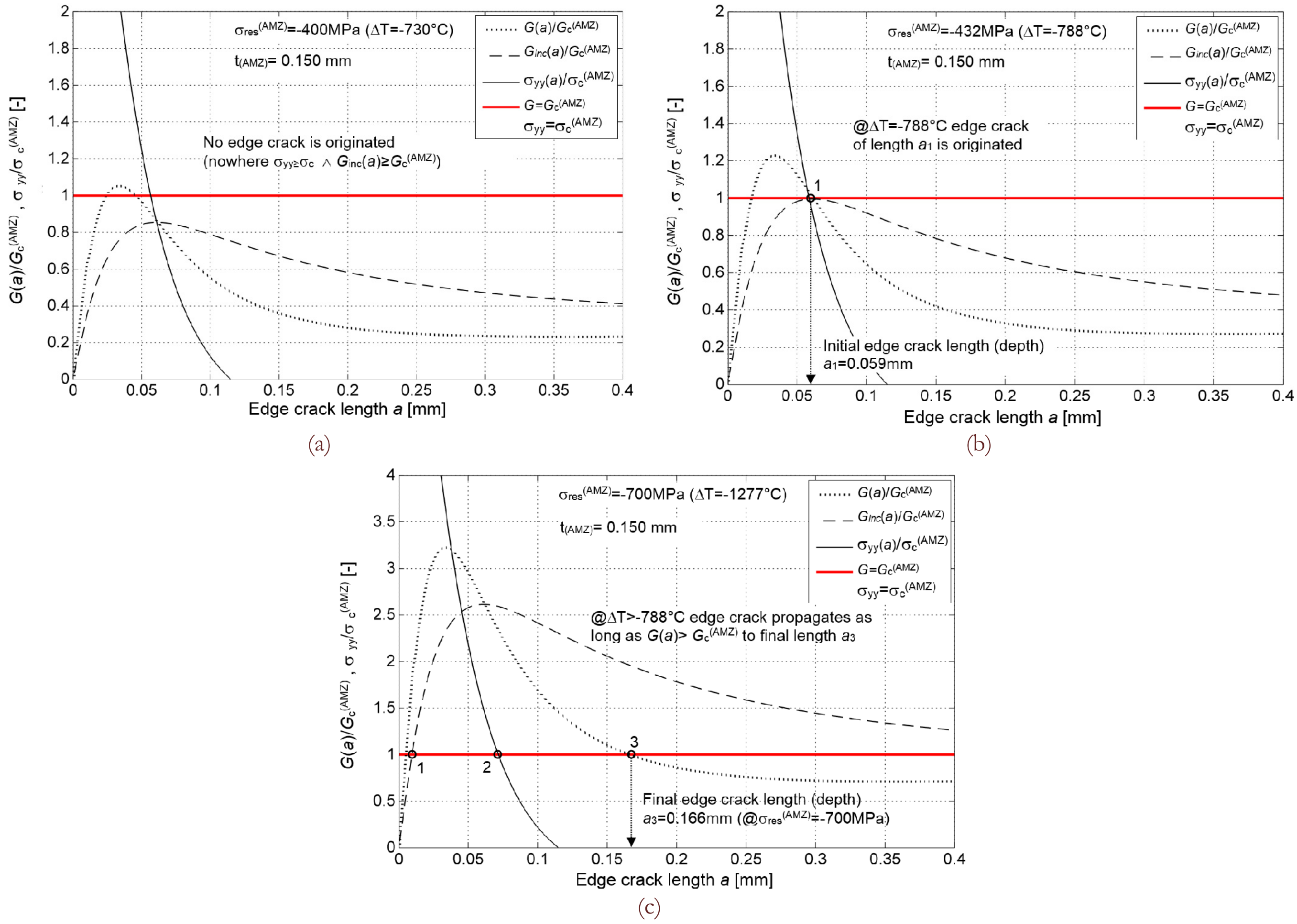

Figure 4. The dimensionless incremental energy release rate $G_{inc}(a)/G_c^{(AMZ)}$, the dimensionless energy release rate $G(a)/G_c^{(AMZ)}$ and the dimensionless tensile stress $\sigma_{yy}(a)/\sigma_c^{(AMZ)}$ for $t_{(AMZ)}$=0.15mm and compressive residual stress in AMZ layer: a) $\sigma_{res}^{(AMZ)}$=-400MPa, b) $\sigma_{res}^{(AMZ)}$=-432MPa, c) $\sigma_{res}^{(AMZ)}$=-700MPa

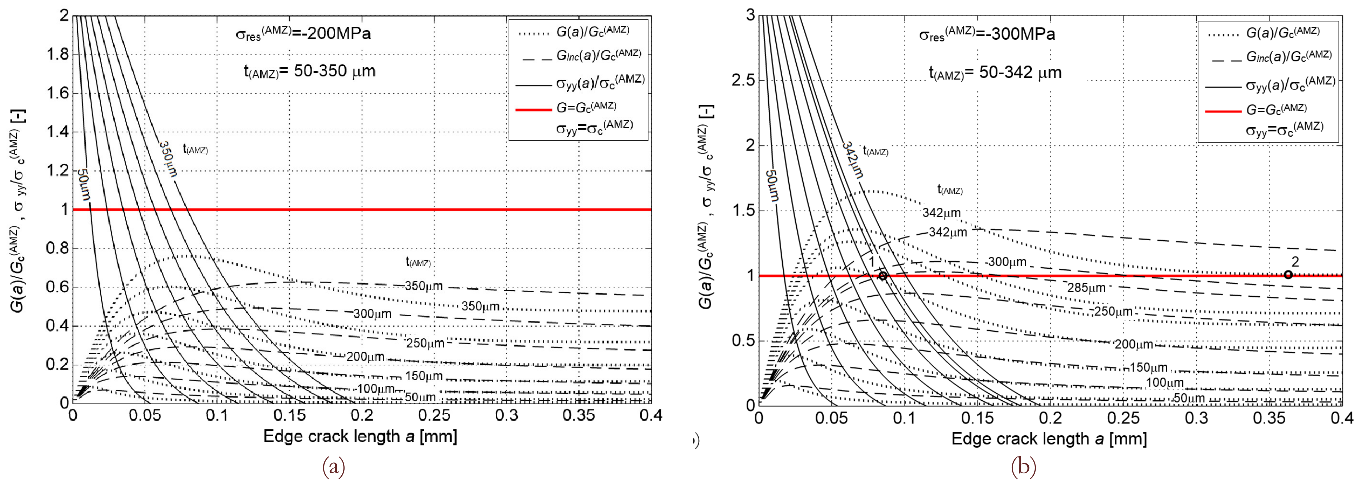

Figure 5. The dimensionless incremental energy release rate $G_{inc}(a)/G_c^{(AMZ)}$, the dimensionless energy release rate $G(a)/G_c^{(AMZ)}$ and the dimensionless tensile stress $\sigma_{yy}(a)/\sigma_c^{(AMZ)}$ for different thicknesses of the compressive AMZ layer at certain level of residual compressive stress: a) $\sigma_{res}^{(AMZ)}$=-200MPa, b) $\sigma_{res}^{(AMZ)}$=-300MPa.

Outlook

The model can be applied to study the combined effect of residual stress and thickness of the compressive embedded layers, which can define lower and upper bounds for the absence or presence of edge cracks in ceramic laminates. Based upon this model, recommendations on layer architectural design can be given, which should prevent the formation of edge cracking in laminate systems designed with internal residual stresses. An extension of this model will be the study of edge crack formation and extension considering the combination of residual stresses and layer thickness as design parameters. In such case, several possibilities for improvement in the mechanical properties of the system (i.e. fracture resistance and strength) are to be expected.

Conclusions

This work presents a novel approach to predict the onset and propagation of surface cracks (namely edge cracks) in ceramic laminates, associated with the residual stresses developed after cooling down from the sintering step. A full parametric 2D finite element model was developed to simulate the initiation and propagation of the edge crack in the compressive layers. The conditions for crack initiation/propagation were assessed using a coupled stress-energy criterion. The analysis only requires the values of the elastic moduli and Poisson's ratio of the layers, the coefficient of thermal expansion, the toughness and the tensile strength of the compressive layer. There is no adjustable parameter and there is no need to assume the presence of surface defects to initiate fracture. It was further found that, for a given thickness of the compressive layer, no edge crack is to initiate for relatively low internal (in-plane) compressive residual stress in the layer. For higher stress values, edge crack may initiate and grow in a stable manner. Finally, for relatively higher stress values, the formation of edge cracks will be followed by the fracture of the entire layer in an unstable fashion. In future work, we will focus on the validation of the obtained results on different real specimens with various thicknesses and levels of residual stresses, aiming to provide guidelines for the fabrication of layered ceramics with controlled surface cracks.

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