Life estimation by varying the critical plane orientation in the modified Carpinteri-Spagnoli criterion

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ABSTRACT. The modified Carpinteri-Spagnoli (C-S) criterion is a multiaxial high-cycle fatigue criterion based on the critical plane approach. According to such a criterion, the orientation of the critical plane is linked to both the averaged directions of the principal stress axes and the fatigue properties of the material. The latter dependence is taken into account through a rotational angle, δ. Then, the multiaxial fatigue strength estimation is performed by computing an equivalent stress amplitude on the critical plane. In the present paper, some modifications of the original δ expression are implemented in the modified C-S criterion. More precisely, such modified expressions of δ depend on the ratio between the fatigue limit under fully reversed shear stress and that under fully reversed normal stress (in accordance with the original expression), and can be employed for metals ranging from mild to very hard fatigue behaviour. Some experimental data available in the literature are compared with the theoretical results in order to verify if the modified expressions are able to improve the fatigue strength estimation capability of the modified C-S criterion.

KEYWORDS. Constant amplitude loading; Fatigue lifetime prediction; Modified C-S criterion; Multiaxial high-cycle fatigue; Critical plane.

INTRODUCTION

In the high-cycle fatigue related to a linear-elastic material, several criteria available in the literature to assess fatigue strength are based on the so-called critical plane approach. This approach takes into account the crack nucleation and early growth mechanisms experimentally observed during cyclic loading. According to such criteria, fatigue failure assessment is performed on a specific plane (the critical plane) within the test specimen or component.

The above criteria are characterized by different rules suitable to define the orientation of the critical plane but, for all these criteria, the fatigue life assessment is carried out by employing a combination of stresses acting on the critical plane itself [1] (see also the review on the critical plane and other approaches to multiaxial fatigue, published in Ref. [2]). For instance, several researchers define the critical plane as the plane where amplitude or some stress component or a combination of them exhibits a maximum value [3-6]. Alternatively, the position of the critical plane may be correlated with that of the principal stress directions by using appropriate weight functions [7].

It is important to highlight that the above definitions represent just some of those reported in the literature (a general account on the critical plane orientation is given in Ref. [8]).
Among the critical plane fatigue criteria, the modified Carpinteri-Spagnoli (C-S) criterion [9], a simplified version of the original C-S criterion [10], correlates the critical plane orientation with the weighted mean directions of the principal stress through an off-angle, δ (which is regarded to be dependent on the ratio between the fatigue limit under fully reversed shear and that under fully reversed normal stress, $\tau_{af,-1}/\sigma_{af,-1}$). Then, the multiaxial fatigue assessment is performed by using a nonlinear combination of the equivalent normal stress amplitude and the shear stress amplitude acting on the critical plane (see Ref. [11] for a general account of the criterion).
In accordance with the original idea developed by Carpinteri et al. [9, 10], Łagoda et al. [12] have recently proposed some modifications to the original δ expression which, in limit conditions, are pertinent to both mild and very hard metals.
The goal of the present paper is to implement the different expressions of the rotational angle δ in the modified C-S criterion. In order to verify whether the above expressions are able to improve the fatigue lifetime estimation capability of the criterion, some experimental data available in the literature [13-17] are examined.

CRITICAL PLANE ORIENTATION AND FATIGUE LIFE EVALUATION

The high-cycle multiaxial fatigue criterion, known as the modified Carpinteri-Spagnoli (C-S) criterion [9], is a simplified version of the original one proposed in Ref. [10]. In particular, the modifications are related to a simplified weighting procedure to determine the averaged principal stress axes, and to the effect of the non-zero normal mean stress on the fatigue limit.
Fig. 1 summarizes how to use the modified C-S criterion to estimate fatigue lifetime of structural components failing in high-cycle fatigue regime.

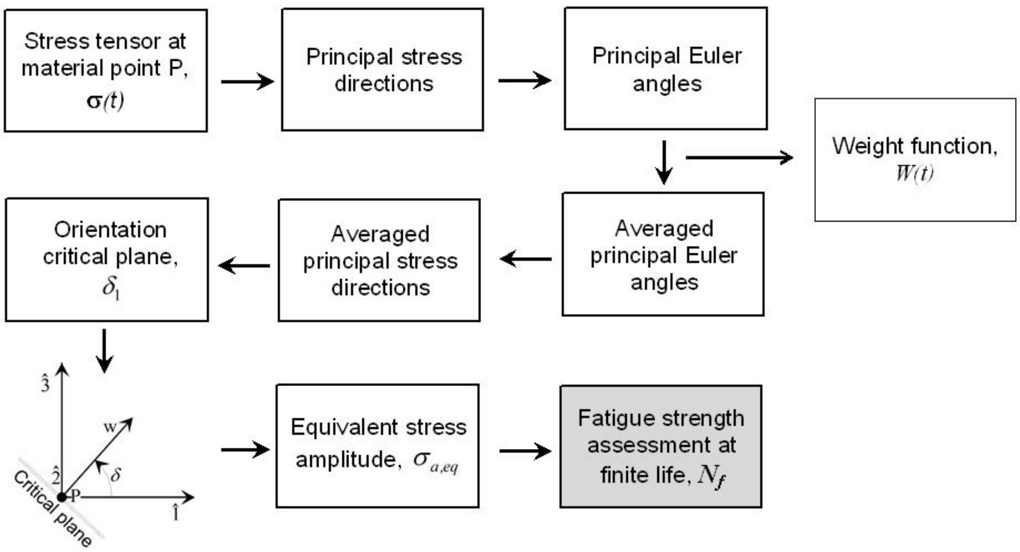

Figure 1: Graphical representation of the modified C-S criterion.

In particular, from the stress state at a material point P, the averaged directions of principal stress axes can be determined on the basis of their instantaneous directions by means of the averaged values of the principal Euler angles. The orientation of the critical plane is linked to the above averaged directions through the rotational angle δ:

$$\delta_1 = 3/2\left[1-\left(\tau_{af,-1}/\sigma_{af,-1}\right)^2\right]\ 45° \quad (1)$$

Then, the fatigue strength is assessed through an equivalent stress amplitude expressed by a quadratic combination of the equivalent normal stress amplitude ($N'_{a,eq}$) and the shear stress amplitude (C_a) acting on the critical plane. Finally, the number of loading cycles to failure, N_f, can be found by solving the following equation through an iterative procedure:

$$\sqrt{\left(N'_{a,eq}\right)^2 + \left(\frac{\sigma_{af,-1}}{\tau_{af,-1}}\right)^2 \left(\frac{N_f}{N_0}\right)^{2m} \left(\frac{N_0}{N_f}\right)^{2m*} \left(C_a\right)^2} = \sigma_{af,-1}\left(\frac{N_f}{N_0}\right)^m \tag{2}$$

where N_0 is the reference number of loading cycles (for example $N_0 = 2 \cdot 10^6$), and m and $m*$ are the slopes of S-N curve for fully reversed normal and shear stress, respectively.

Recently, Łagoda et al. [12] have proposed some modifications to the original δ expression. In accordance with the idea originally developed by Carpinteri et al. [9-10] to assume that δ is function of the ratio $\tau_{af,-1}/\sigma_{af,-1}$ (such an expression can be employed for metals ranging from mild to very hard fatigue behaviour), the relationships reported in Ref. [12] are the following:

$$\delta_2 = \frac{9}{8}\left[1-\left(\tau_{af,-1}/\sigma_{af,-1}\right)^4\right] 45° \tag{3}$$

$$\delta_3 = \frac{3\sqrt{3}}{3\sqrt{3}-1}\left[1-\left(\tau_{af,-1}/\sigma_{af,-1}\right)^3\right] 45° \tag{4}$$

$$\delta_4 = \frac{3\sqrt{3}}{3\sqrt{3}-3}\left[1-\left(\tau_{af,-1}/\sigma_{af,-1}\right)\right] 45° \tag{5}$$

$$\delta_5 = \frac{3}{\left(\sqrt{3}-1\right)^2}\left[1-\left(\tau_{af,-1}/\sigma_{af,-1}\right)\right]^2 45° \tag{6}$$

EXPERIMENTAL VALIDATION

In the present paper, the different relationships of the rotational angle δ previously described are implemented in the modified C-S criterion in order to verify whether they are able to improve the above criterion in terms of lifetime estimation of some experimental test results available in the literature.

The examined data are related to samples made of 30CrNiMo8 Steel [13,14], 6082-T6 Aluminum Alloy [15,16] and S335J0 Alloy Steel [17] subjected to synchronous, sinusoidal, in-phase loading (with zero and non-zero mean value). The relevant mechanical properties for each examined material are reported in Tab. 1.

Material	σ_u [MPa]	$\sigma_{af,-1}$ [MPa]	m [-]	$\tau_{af,-1}$ [MPa]	$m*$ [-]
30CrNiMo8 Steel	1014	427.37	-0.13	371.52	-0.04
6082 - T6 Aluminium Alloy	290	152.83	-0.11	87.90	-0.15
S355J0 Alloy Steel	611	276.58	-0.15	183.70	-0.09

Table 1: Static and fatigue properties for each examined material.

The mean square-root error method [12] is applied to the statistical analysis of the fatigue lifetime results determined by using the modified C-S criterion. In particular, the value of the root mean square logarithmic error is computed as follows:

$$E_{RMS} = \sqrt{\frac{\sum_{i=1}^{n} \log^2\left(N_{f,\exp}/N_{f,cal}\right)}{n}} \tag{7}$$

where n is the total number of data, $N_{f,\exp}$ is the experimental multiaxial fatigue life, and $N_{f,cal}$ is the theoretical multiaxial fatigue life determined by considering Eqs (1; 3-6). The mean square error T_{RMS} is given by: $T_{RMS} = 10^{E_{RMS}}$.
For the different examined materials, Fig. 2 (a), (b) and (c) represents the mean square error obtained for the five expressions of angle δ (from δ_1 to δ_5).

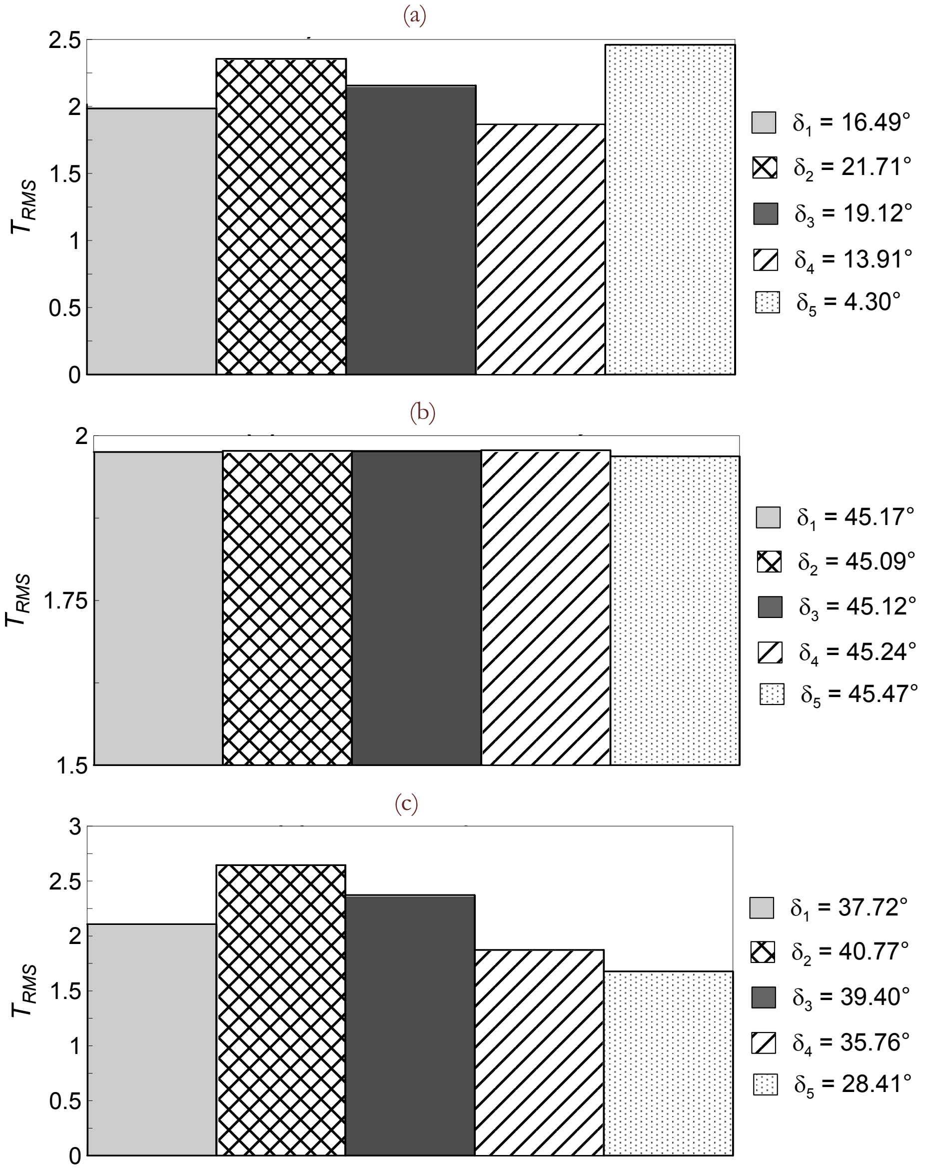

Figure 2: Mean square error related to: (a) 30CrNiMo8 Steel; (b) 6082 - T6 Aluminum Alloy ; (c) S335J0 Alloy Steel.

A good agreement between experimental and theoretical results is in general observed, since the value of T_{RMS} is lower than 3 (note that if all the calculated results fell within the scatter band 2, the value of T_{RMS} would be equal to 2).
Moreover, the analysis of the results in terms of the mean square error for the examined materials indicates that:

a) for 30CrNiMo8 Steel, higher accuracy is gained for the orientation of the critical plane computed by means of δ_4. Note that, by implementing Eq.(5) in the modified C-S criterion, the value of T_{RMS} decreases to 6% in comparison to that determined by applying the δ_1 expression;
b) for 6082 - T6 Aluminum Alloy, the same accuracy is deduced by using the five different δ expressions, since the value of δ is essentially the same;
c) for S335J0 Alloy Steel, the most accurate result is provided by using the δ_5 expression to determine the critical plane, with a decrease of the T_{RMS} value up to 20.4% with respect to that deduced by computing the critical plane orientation through Eq.(1).

Therefore, the implementation of the δ relationships (proposed by Łagoda) in the modified C-S criterion yields, only for materials characterized by fatigue limit ratio typical of hard and very hard metals, fatigue lifetime results different from those determined through the original δ expression. In particular, the δ_4 and δ_5 expressions, respectively for 30CrNiMo8 Steel and for S335J0 Alloy Steel, provide better results than those deduced by employing the other relationships.

Conclusions

In the present paper, the orientation of the critical plane, linked to the averaged principal stress directions, is computed by taking into account different expressions of the rotation angle δ. In particular, such relationships have been implemented in the modified C-S criterion in order to estimate the fatigue lifetime by varying the critical plane orientation. The comparison with some experimental data related to stress-controlled fatigue tests of specimens under biaxial loading appears to be satisfactory. In particular, better estimations in terms of fatigue life are obtained for experimental data related to hard metals by using some of the modified δ expressions (instead of the original one).

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