Multi-challenge aspects in fatigue due to the combined occurrence of multiaxiality, variable amplitude loading, and size effects

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ABSTRACT. The three major issues which contribute to uncertainties in fatigue life estimations are the limited transferability of material data obtained in a laboratory for describing the fatigue behavior of components, the lack of damage accumulation rules to realistically take into account the effects of various load sequences, and the uncertainty in the assessment of multiaxial stress states. The three issues contribute to life prediction errors in a balanced way. Some modeling effort was spent in joining solution proposals in a unifying short crack model. The actual state is presented in the paper together with some ideas for further improvement as well as simplification for promoting the model's acceptance in practical applications.

KEYWORDS. Multiaxial fatigue; Short crack; Critical plane.

INTRODUCTION

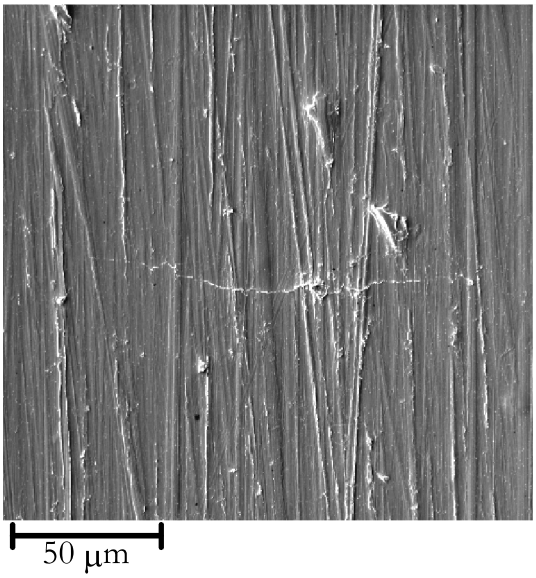
For the stage of technical fatigue crack initiation conventional approaches like the local strain approach are applied which do not explicitly refer to the physical process of defect growth. This stage is treated as a black box where the failure process is disguised behind strain or stress life curves, damage accumulation rules, and multiaxial failure hypotheses. In such simulations, three major issues contribute to uncertainties in fatigue life estimations, the limited transferability of material data obtained in a laboratory for describing the fatigue behavior of components, the lack of damage accumulation rules to realistically take into account the effects of various load sequences, and the uncertainty in the assessment of multiaxial stress states.

In a multi-scale modeling of the fatigue process, the next smaller scale of observation is the short crack growth regime. A large amount of research was dedicated to the investigation and accompanying modeling of short crack growth behavior with the aim to reduce the inaccuracies of conventional fatigue life estimation approaches. The accuracy of damage accumulation rules under variable amplitude loading was significantly enhanced by modeling the sequence-dependent opening and closure of short cracks. Origins of limited transferability of material data may be identified in the size effect, the roughness effect, or the presence of residual stresses. Some of these influences can be well explained using a short crack approach. For example, the non-homogeneous stress field in notches affects the crack driving force. Considerable effort has been dedicated to model the statistical size effect, too. From the viewpoint of a short crack approach, the probability of exposure of a strength-reducing defect increases with the surface of the highly stressed material. Finally, the short crack approach was introduced for the assessment of multiaxial fatigue.

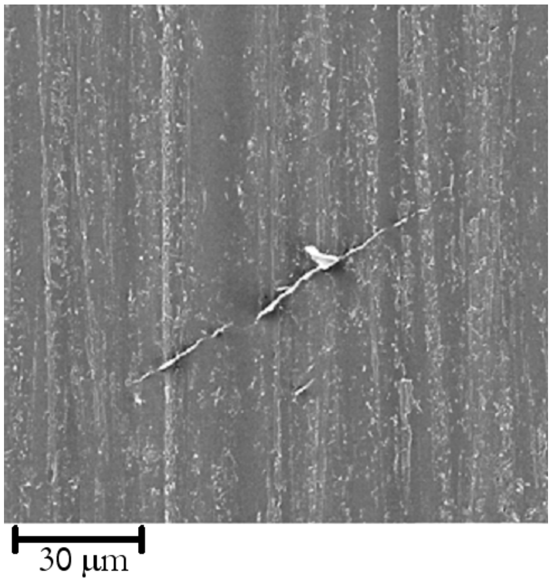
The three mentioned issues contribute to life prediction errors in a balanced way. Some modeling effort was spent by Hertel and Vormwald [1,2] during the last years in joining solution proposals in a unifying short crack model. The actual state is presented together with some ideas for further improvement and simplification for promoting the model's acceptance in practical applications.

MULTIAXIAL FATIGUE CRACK INITIATION

The macroscopic appearance of early fatigue crack initiation is similar under uniaxial, proportional, or non-proportional loading. In all cases short cracks initiate at microscopically small defects. A stage of stable fatigue crack growth follows. The crack selects a plane of growth which is usually termed critical plane. Therefore, the critical plane approaches to multiaxial fatigue assessment are among the most preferred approaches. Variants of this approach are explained in the standard text book of Socie and Marquis [3]. As an example, Fig. 1 shows fatigue cracks initiated under uniaxial and severely non-proportional multiaxial loading. The crack growth has been monitored by Hoffmeyer [4] who applied the replica technique. A part of this investigation has been published earlier in references [5] and [6] where more details can be found.



a)



b)

Figure 1: Appearance of short fatigue cracks in AW5083 (also nominated as AlMg4.5Mn or Al5083), a) uniaxial loading with $\varepsilon_a = 0.3\%$, b) multiaxial non-proportional pseudo-random strain sequence, straight-line cumulative frequency distribution $\varepsilon_{a,\max} = 0.346\%$, $\gamma_{a,\max} = 0.6\%$ and a phase shift of 90° between the normal and the shear strain signal, Hoffmeyer [4].

Challenge 1: Crack driving force

It suggested itself to model the mechanisms by means of fracture mechanics [7-19]. Naturally initiating cracks require stress and strain amplitudes of a magnitude that usually prevents the application of linear elastic fracture mechanics. A crack driving force from the collection of elastic-plastic fracture mechanics is selected. The first challenge is to define a reasonable and applicable formulation. Strain based intensity factors and variants of the cyclic ΔJ -integral are used. Here, some results are reported which adapted a formulation based on approximation formulas assumed to provide an estimate of the ΔJ -integral.

$$\Delta J_{\mathrm{I,eff}} = 2\pi Y_{\mathrm{I}}^2 \Delta W_{x,\mathrm{eff}} a \tag{1}$$

$$\Delta J_{\mathrm{II,eff}} = \pi / (1+\nu) Y_{\mathrm{II}}^2 \Delta W_{xz} U_{\mathrm{eff}} a \tag{2}$$

$$\Delta J_{\mathrm{III,eff}} = \pi Y_{\mathrm{III}}^2 \Delta W_{xy} U_{\mathrm{eff}} a \tag{3}$$

These formulas are supposed to be valid for semi-circular surface cracks. The *x*-coordinate is perpendicular to the crack area. Besides geometry factors Y for the three opening modes, the estimates rely on a proportionality between the direction-related portions of the strain energy density range, ΔW_{ij}, and the corresponding mode-related ΔJ -integral. A numerical check of the accuracy of this assumption is pending.

Challenge 2: Effective ranges

From uniaxial loading it is known that cracks only grow when their crack flanks do not touch, see e.g. [20]. Little is known concerning crack closure under multiaxial loading. An example has been presented by Hoffmeyer [4]. A thin-walled tube from steel S460N was tested under stain control with a butterfly strain sequence, Fig. 2, two shear strain cycles, γ, were applied during one normal strain cycle, ε. Thirty-three surface replicas of a naturally initiated crack were taken during a butterfly cycle at the γ-ε combinations indicated by dots in Fig. 2. The snapshots are grouped together in an animation provided by a link.

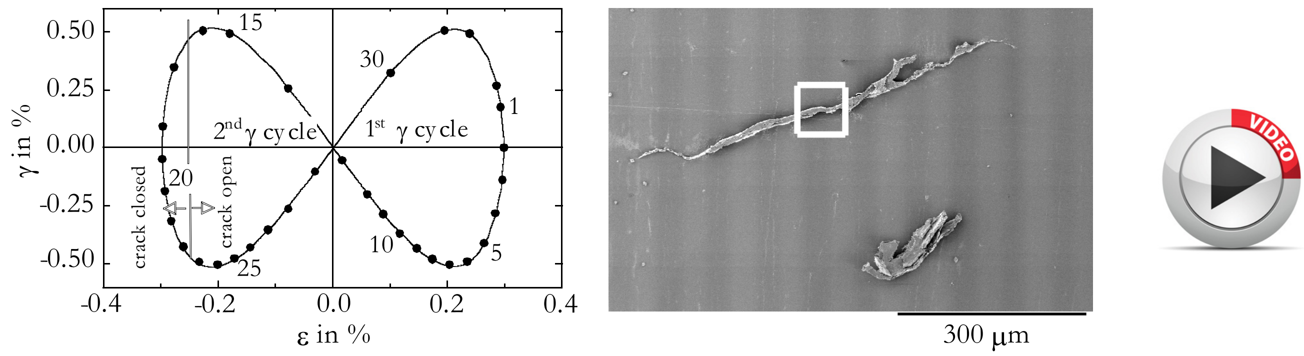


Figure 2: Strain sequence, indicating (by dots) the γ-ε combinations of replicas, left, and inspection area of crack opening displacement evaluation, right, Hoffmeyer [4].

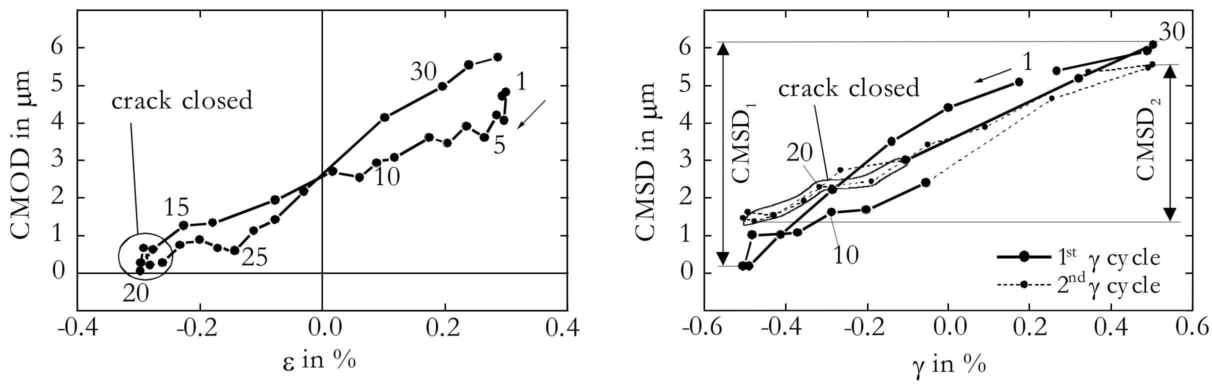


Figure 3: Crack mouth opening and sliding displacements, Hoffmeyer [4].

The crack mouth opening and sliding displacements, CMOD and CMSD, have been measured. In Fig. 3 the local crack flank displacements (taken in the middle of the section shown by a white square in Fig. 2 right) are plotted over the applied global strains. The crack flanks are in contact when the minimum normal strain is applied. The vertical line in Fig. 2 gives an estimate of the crack opening and closure strain. Crack flank sliding occurs while crack flanks are in contact, Fig. 3 right. However, sliding is reduced, $CMSD_2$, compared to a shear cycle without any contact, $CMSD_1$. Such observations of crack closure led to a phenomenological description of effective ranges in Eq. (1) to (3). For the mode I loading portion the crack opening and closure global strain ($\varepsilon_{x,\mathrm{op}}$ and $\varepsilon_{x,\mathrm{cl}}$ are assumed to coincide) are modelled by first calculating an opening stress with

$$
\begin{aligned}
&\sigma_{x,\mathrm{op}} = \sigma_{x,\max}(A_0 + A_1 R \sigma_{\mathrm{eqv,max}} / \sigma_{\mathrm{max,eff}}) \text{ for } R<0 \\
&A_0 = 0.535\cos\left((\pi\sigma_{\mathrm{max,eff}})/(2\sigma_{\mathrm{F}})\right) \\
&A_1 = 0.344(\pi\sigma_{\mathrm{max,eff}})/(2\sigma_{\mathrm{F}}) \qquad (4) \\
&\sigma_{\mathrm{max,eff}} = \sqrt{\sigma_{x,\max}^2 + 3(\tau_{xy}^2 + \tau_{xz}^2)} \\
&\sigma_{\mathrm{eqv,max}} = \sqrt{(\sigma_{x,\max}-\sigma_y)^2 + (\sigma_{x,\max}-\sigma_z)^2 + (\sigma_y-\sigma_z)^2 + 6(\tau_{xy}^2+\tau_{xz}^2+\tau_{yz}^2)} / \sqrt{2}
\end{aligned}
$$

where the x-coordinate is the crack area normal direction. The stress tensor components have to be provided in this coordinate system. Next, the crack opening strain is determined in the ascending σ_x-ε_x path. The effective crack driving force is determined on the descending σ_x-ε_x path,

$$\Delta W_{x,\mathrm{eff}} = \int_{\varepsilon_{x,\max}}^{\varepsilon_{x,\mathrm{cl}}} \left(\sigma_x - \sigma_{x,\max}\right) \mathrm{d}\varepsilon_x \,. \tag{5}$$

The effective ranges associated with shear modes are estimated by defining an efficiency factor with

$$U_{\mathrm{eff}} = \ln\left(\cos\left(\left(\pi(\Delta\tau/2 - \tau_{\mathrm{fric}})\right) / \left(2(\tau_{\mathrm{F}} - \tau_{\mathrm{fric}})\right)\right)\right)^{-1} / \ln\left(\cos\left(\left(\pi(\Delta\tau/2)\right) / \left(2\tau_{\mathrm{F}}\right)\right)\right)^{-1}, \tag{6}$$

where

$$\tau_{\mathrm{fric}} = \left\langle \tau_{\mathrm{act}} - \mu\sigma_{\mathrm{n,max}} \right\rangle \tag{7}$$

is a model parameter requiring two fitting constants, τ_{act} and μ, and $\sigma_{n,max}$ is the maximum normal stress during a shear cycle. These remarks on the crack closure phenomenon and the corresponding effective ranges should have made clear that the problems is away from being solved generally and provides another challenge for future research.

Challenge 3: Definition of a damaging event – or a cycle

Most procedures apply rainflow counting of stress or strain components acting on the critical plane. There is hardly an alternative for defining a cycle although the physical basis of the rainflow algorithm is lost in non-proportional loading. A desirable approach would yield an incremental crack growth for an infinitesimal variation of the crack driving force. Such an approach has been proposed for example by Lu and Liu [21] for uniaxial loading. For multiaxial loading some trials have been published, e.g. [22], to use plastic work as the responsible quantity for incrementally increasing the fatigue damage. The models have not yet achieved a wide acceptance.

Challenge 4: Stress-strain paths

The input for the cycle identification and counting must be provided by a calculation of components of the stress and strain tensor as a function of time. Solving this task requires application of a cyclic plasticity model. Contrary to a uniaxial case, the differential equations of incremental plasticity have to be solved. These models should address two phenomena which emerge in non-proportional loading: multiaxial ratcheting and non-proportional hardening. Here, a model suggested by Döring et al. [23] was used. Fig. 4 shows a comparison of measured and calculated stresses for a strain controlled butterfly path. An animation showing the calculated stress path together with the kinematics of the yield surface is provided by link. The accuracy is better than acceptable; the numerical expense is enormous.

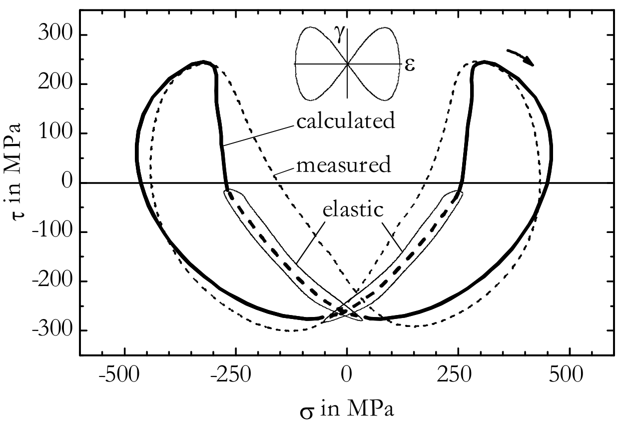


Figure 4: Measured and calculated stress response, strain controlled butterfly loading of a thin-walled tube, S460N, Döring et al. [23].

Challenge 5: Mixed mode hypothesis

The combined effect of mode-related crack driving forces on the crack increment has to be evaluated. Little is known concerning mixed mode hypotheses for non-proportional fatigue [24]. Generally, the hypothesis has to specify the crack advance direction and the increment. In the crack initiation stage kinking in the critical plane is excluded and therefore only the increment must be calculated. It is not much more than a working hypothesis to simply add contributions of each mode according to

$$\mathrm{d}a\,/\,\mathrm{d}n = C\cdot\Delta J_{\mathrm{I,eff}}^{\mathrm{m}} + C\cdot\Delta J_{\mathrm{II,eff}}^{\mathrm{m}} + C\cdot\Delta J_{\mathrm{III,eff}}^{\mathrm{m}}\,, \tag{8}$$

where *m* is the exponent of a power-law type crack growth rate formulation in terms of the ΔJ-integral.

TRANSFER OF MATERIAL DATA FOR DESCRIBING THE FATIGUE BEHAVIOR OF COMPONENTS

The information gained by investigating material's fatigue behavior under multiaxial fatigue must be made available for calculating fatigue lives of components. Several tasks have to be fulfilled before arriving at a result. Generally, a local stress or strain based approach is used. This implies the assumption that the life of a component can be determined from the failure life of a material element at the location of severest local stress and strain history.

Challenge 6: Identification of critical locations

It is not possible to identify the critical location only on the basis of quasi-static calculations of stresses and strains in the component, even after having gained the solutions for all loading cases. The fatigue live calculation for possibly very many locations has to be performed. The shortest of all lives obtained will determine the component's life.

Challenge 7: Calculation of local stress-strain time histories

The method for calculating local stress and strain sequences might be the application of the finite element technology. However, the computational costs of such an approach are still too high for practical application. All actual software packages in the field heavily rely on approximations of the local elastic-plastic stress-strain histories on the basis of the knowledge of a hypothetical or pseudo linear elastic local stress histories, ${}^{e}\sigma_{ij}(t)$. For all unit load cases, L_k=1, the transformation factors, $(c_{ij})_k$, are calculated. The local pseudo-elastic stress time sequences are calculated by superposition,

$${}^{e}\sigma_{ij}(t) = \sum_{k=1}^{l}\left(c_{ij}\right)_k L_k(t) \tag{9}$$

Köttgen et al. [25] suggested two approaches, the pseudo stress and the pseudo strain based approach. Both approaches are based on a two-step procedure. First, the relation between the loads and local stresses or plastic strains has to be established. Second, the latter are related to its local counterpart. The term "relation" means application of an incremental plasticity model. In the pseudo stress approach loads are expressed in terms of pseudo stresses according to Eq. (9), in the pseudo strain approach loads are expressed as pseudo strains.
Buczynski and Glinka's [26] extended Neuber's rule to non-proportional loading. The authors assume that the total of the distortional strain energy in an increment of loading is equal for elastic-plastic and linear elastic deformation. More precisely, the assumed identity is formulated for each component of the deviatoric stress and strain tensors individually.
Hertel et al. [27] compared the results of various approaches for a notched shaft under non-proportional tension-compression and torsion. The results for a butterfly-type loading are redrawn in Fig. 5. The accuracy of all approaches is comparable. The differences in predicted stress-strain paths are in the same order of magnitude as differences between measured and FE-calculated paths. Some more comparisons can be found in [28].

Challenge 8: Size effects

In the literature on size effects in fatigue several types of size effect are distinguished [29]. Most important are the types taking into account the stress gradient and the highly stressed surface. Although fracture mechanics is the favorite method for describing the stress gradient effect, the required solutions for the crack driving forces in non-homogeneous stress fields have not yet been developed. In the uniaxial case, only very sharp gradients have a considerable effect on crack initiation lives [29]. Here, gradient effect has not yet been taken into account.
Fatigue life calculations based on defect growth considerations require the definition of an initial defect size. The initial crack depth for starting the simulation, a_0, has to be determined. It is chosen in such a way that the life calculation model provides a realistic strain life curve of the material under consideration. The initial crack depth is regarded as a Weibull-distributed random variable. The probability of finding an initial crack with a depth larger then a_0 is written as

$$P(a_0) = \exp(-(a_0 / a_\kappa)^\kappa) \quad (10)$$

The distribution parameter a_κ and the Weibull-exponent κ are determined based on a statistical evaluation of the scatter of the initial crack depths from scatter of strain-life data. The initial crack depth for a component with any highly stressed surface A can be calculated by

$$a_0 = a_{0,\text{ref}}(A / A_\text{ref})^{1/\kappa} \quad (11)$$

Here $a_{0,\text{ref}}$ and A_ref are the initial crack depth and the highly stressed surface of the reference or material specimen. The variables a_0 and A are the initial crack depth and the highly stressed surface of the structure under investigation. The component's fatigue life therefore differs from material specimen's fatigue for identical local stress and strain histories.

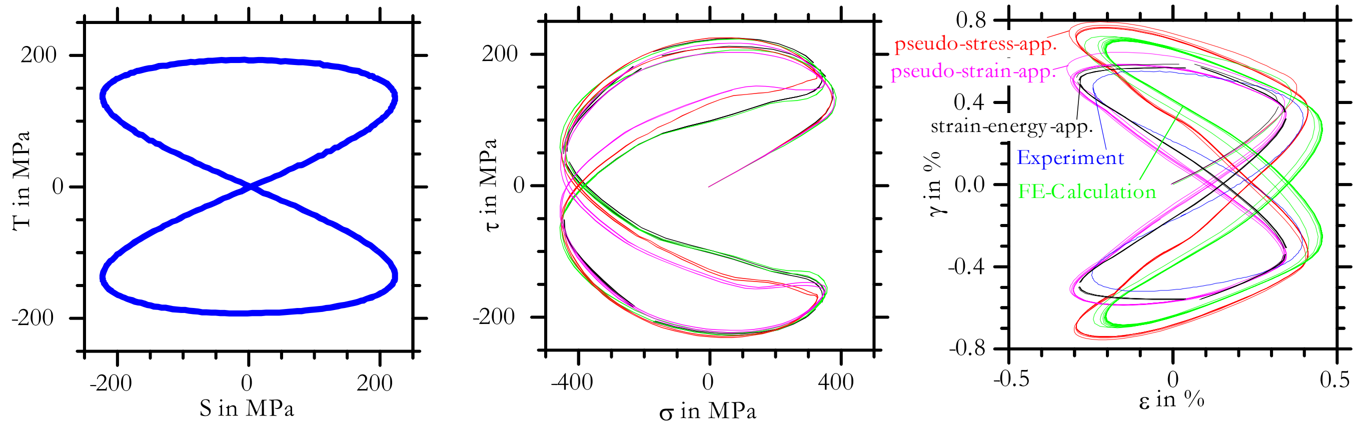


Figure 5: Local, notch root stress, σ−τ, and strain, ε−γ, states for a notched shaft under load controlled butterfly loading with tension-compression (nominal stress S) and torsion (nominal stress T), S460N, Hertel et al. [27].

SEQUENCE EFFECTS IN VARIABLE AMPLITUDE LOADING

Two major sources of sequence effect have been identified, either cyclic plastic deformation or geometric issues like a changing geometry of the structure during the fatigue process [30]. Some of the sequence effects are severe. Mean stress re-arrangement and plasticity induced crack closure as well as the continuous decrease of the technical endurance limit very strongly influence variable amplitude fatigue lives.

Challenge 9: Sequence effects due to plasticity induced crack closure

In the section on challenge 2 the determination of effective ranges for constant amplitude loading has been described. The crack opening strain $\varepsilon_{x,op}$ (or $\varepsilon_{x,cl}$, respectively) discriminates effective and non-effective portions of a cycle. Under variable amplitude loading the crack opening strain is considered to depend on the load sequence. Two main cases are distinguished. If the constant amplitude crack opening strain of a cycle is smaller than the crack opening strain of previous cycles the crack opening strain is set to its constant amplitude value. This case is typical for large cycles following small cycles. If, however, the constant amplitude crack opening strain of a cycle is greater than the crack opening strain of previous cycles, the effective range of the actual cycle is computed using the old, lower crack opening strain. This case is typical for small cycles following large cycles. In this case effective ranges and crack growth increments of the proceeding small cycles are larger than they would be under constant amplitude conditions. An accelerating sequence effect is modeled. The small cycles following a large cycle continuously restore their constant amplitude crack opening strain level. The algorithm is described in more detail in reference [1]. It relies on rainflow cycle counting of the strain sequence.

Challenge 10: Sequence effects due to increasing crack size

A major issue in the assessment of variable amplitude loading is to realistically account for the damage contribution of load cycles with amplitudes smaller than the constant amplitude endurance limit. A fracture mechanics based assessment offers a natural solution of this problem. The threshold condition discriminates between growing and non-growing cracks.

With the increasing crack length more and more smaller cycles contribute to crack growth. Here, crack growth is calculated only for cycles for which the crack driving force is larger than its threshold. Special attention must be paid to the fact that a short crack (SC) growth threshold is smaller than the long crack (LC) growth threshold. Its dependence on the crack depth a is described by an empirical equation,

$$\Delta J_{\text{eff,th,SC}} = \Delta J_{\text{eff,th,LC}} \cdot a \,/\, (a + l^*) \qquad (12)$$

The size of l^* is determined by the fatigue strength of the material and the long crack growth threshold.

Results

The notes on the challenging tasks listed above should have made clear that accurate life prediction results cannot be expected, yet. Too many assumptions and simplifications had to be introduced. Nevertheless, comparisons of experimentally and numerically determined crack initiation lives have been performed. The results for the most challenging cases of notched components under multiaxial variable amplitude loading are revisited, Fig. 6.

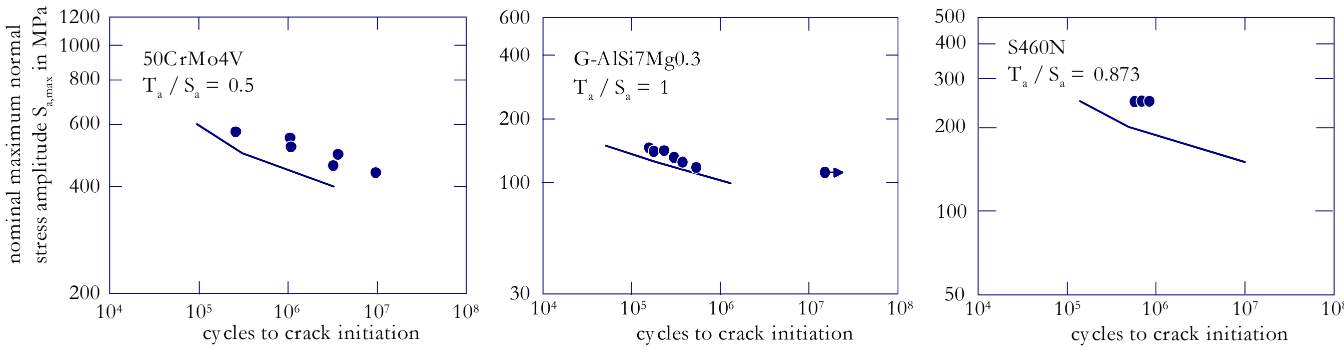


Figure 6: Experimentally ([1, 31]) and numerically ([1, 2]) determined crack initiation lives of shouldered shafts under non-proportional variable amplitude loading, nominal normal stress S and nominal shear stress T, phase shift 90°, pseudo random sequence with Gaussian cumulative frequency distribution.

A tendency of underestimating fatigue lives is observed. It might be attributed to the assessment of accelerating sequence effects. Besides, the overall acceptable accuracy is certainly due to annihilation of errors gathered by imperfectly coping with the ten challenges.

Conclusion

In the assessment of fatigue of components under multiaxial variable amplitude loading a list of challenges has been identified. Any method is obliged to provide answers to all challenges. The list is incomplete. For example, the role of the inhomogeneous microstructure has not been addressed in this paper. However, it might have special influence. Many materials are anisotropic featuring special weak planes of preferred crack growth.

The actual experience leads to a ranking of the challenges with respect to their importance for the accuracy of the calculated lives. The mixed mode hypothesis should be appropriate – and at the same time it is the theory with weakest support for non-proportional loading. The crack closure plays a dominant role in connection with mixed mode and variable amplitude sequence effects. And the interaction with size effect assessment methods is highly speculative. On the other hand, the models for describing cyclic plastic deformation and for local strain estimation are well validated.

A second ranking occurs to be performed with respect to simplifying the algorithms in order to accelerate the computation and promote acceptance. Here, the models for describing cyclic plasticity rank first. Most desirable would be an acceleration of the search of the critical plane orientation. And finally, the tracking of sequence effects calls for speeding up.

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