Growth of inclined fatigue cracks using the biaxial CJP model

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ABSTRACT. The CJP model of crack tip stresses is a modified version of the Williams crack tip stress field which takes account of simplified stress distributions that arise from the presence of a zone of plastic deformation associated with the crack flanks and crack tip, and that act on the elastic field responsible for driving crack growth. The elastic stress field responsible for crack growth is therefore controlled by the applied loading and by the induced boundary stresses at the interface with the plastic zone. This meso-scale model of crack tip stresses leads to a modified set of crack tip stress intensity factors that include the resultant influence of plastic wake-induced crack tip shielding, and which therefore have the potential to help resolve some long-standing controversies associated with plasticity-induced closure. A full-field approach has now been developed for stress using photoelasticity and also for displacement using digital image correlation. This paper considers the characterisation of crack growth rate data with the biaxial CJP model, using compact tension specimens that contain inclined cracks at the notch tip with initial angles of 30°, 45° and 60° to the horizontal axis. Significant experimental difficulties are experienced in growing cracks in a biaxial field under uniaxial tensile loading, as the natural tendency of the crack is to turn so that it becomes perpendicular to the maximum principal stress direction. However, crack angle is not an issue in the CJP model which calculates the stress field parallel with, and perpendicular to, the crack plane. These stress components can be rotated into directions comparable with the usual K_I and K_{II} directions and used to calculate stress intensity parameters that should be directly comparable with the standard stress intensity formulations. Another difficulty arises, however, in finding published expressions for K_I and K_{II} for CT specimens with curved or kinked cracks. The CJP model has been successful in achieving a sensible rationalisation of crack growth rate data for the specimens considered in this work, although some observations are not easily explained. Nonetheless, considering the complexity of characterising crack growth rates for cracks with an initial orientation of 30°, 45° or 60° to the horizontal and which subsequently change angle during growth, the results found so far indicate that there is value in further pursuing the CJP approach. The paper introduces future research directions for the CJP model.

KEYWORDS. Biaxial crack tip displacement field; CJP model; William's model; Compact tension specimen; Digital image correlation; Fatigue crack growth rate.

INTRODUCTION

The CJP model of crack tip stresses is a modified version of the Williams crack tip stress field which takes account of simplified stress distributions that arise from the presence of a zone of plastic deformation associated with the crack flanks and crack tip, and that act on the elastic field that drives crack growth [1]. The elastic stress field responsible for crack growth is therefore controlled by the applied loading and by the induced boundary stresses at the interface with the plastic zone. The CJP model is essentially an extension of the crack tip stress model which underpins the use of the classic stress intensity factor K and that uses a 'plastic inclusion' approach for dealing with the stresses induced by local plasticity which is concomitant with a growing fatigue crack [1]. A localised zone of plasticity arises from crack growth mechanisms and essentially blunts the crack and creates a reversed cyclic plastic zone. In addition the compatibility requirement for displacements at the elastic-plastic boundary induces interfacial shear stresses along the crack flanks, along with the possible generation of wake contact stresses. These induced stresses act on the applied elastic stress field at the boundary of the elastic-plastic enclave surrounding the crack. This meso-scale model of crack tip stresses leads to a modified set of crack tip stress intensity factors that include the resultant influence of plastic wake-induced crack tip shielding, and which therefore have the potential to help resolve some long-standing controversies associated with plasticity-induced closure. This full-field approach has been developed for stress using photoelasticity [1] and also for displacement using digital image correlation [2]. The CJP model was initially derived for a uniaxial stress field and was then extended to deal with biaxial loading; the appropriate equations being presented at the Malaga Crack Tip fields conference in 2013 [2]. It is worth noting that Tada and Paris [3] used a similar approach in considering the influence that additional terms in the Westergaard stress function might have on fatigue crack growth when the crack surfaces are not stress-free. Their examples were focussed on specific cases of applied forces, e.g. an internal crack subject to concentrated Mode 1 splitting forces. They did not consider plastic constraint effects and were not focussing on crack tip shielding, but did give some attention to the possible presence of terms which cannot be included in the series expansion of the form given below:

$$Z(z) = \frac{K_0 + K_1 z + K_2 z^2 + \ldots + K_n z^n +}{\sqrt{2\pi z}} \quad (1)$$

In the period between the Malaga and Urbino Crack Tip Fields conferences, an internet-mediated research group has been established between research groups at Plymouth (James, Christopher) and Liverpool (Patterson) in the UK and Jaen (Díaz Garrido) in Spain. The intention has been to characterise the plastic zone size and shape using sophisticated experimental techniques (e.g. thermoelastic stress analysis, electronic speckle pattern interferometry, and digital image correlation) and materials with several strain hardening exponents, and to compare this data with the predictions of the Williams crack tip stress model and the CJP model. As the CJP model contains five terms, the Williams crack tip stress expansion is considered with two terms and with five terms. Preliminary results from this work have indicated that the CJP model gives a closer correlation to the size and shape of the experimentally obtained plastic zones than the Williams model.

The part of the work that will be presented in this paper has been aimed at exploring the characterisation of crack growth rate data with the biaxial CJP model. Crack growth rates have been measured with compact tension specimens that contain inclined cracks at the notch tip with initial angles of 30°, 45° and 60° to the horizontal axis. Significant experimental difficulties are experienced in growing cracks in a biaxial field under uniaxial tensile loading, as the natural tendency of the crack is to turn so that it becomes perpendicular to the maximum principal stress direction. Coupled with this tendency, the tension, bending and buckling stress field in thin compact tension (CT) specimens also lead to changes in crack direction as the crack extends. The CJP model resolves the displacement field around the crack tip to obtain stresses and hence stress intensity factors (K_F, K_R and K_S) parallel with, and perpendicular to, the crack plane. K_F represents the total driving force on the crack, K_R the retarding influences arising from the plastic enclave and K_S reflects the compatibility-induced shear stresses at the elastic-plastic interface along the complete plastic boundary. In the CJP model crack angle is not an issue, and the stress components can be rotated into directions comparable with the usual K_I and K_{II} directions and used to calculate stress intensity parameters that should be directly comparable with the standard stress intensity formulations. Another difficulty arises, however, in finding published expressions for K_I and K_{II} for CT specimens with curved or kinked cracks.

This paper will present experimental crack growth rate data obtained from testing inclined notch specimens at R=0.1 and characterised using the three stress intensity factors derived from the biaxial CJP model, which is given in Eq. (2) [2]:

$$\left|\sigma_y - \sigma_x + 2i\sigma_{xy}\right| = (A_r + i3B_i)z^{-\frac{1}{2}} + (B_r + iB_i)z^{-\frac{3}{2}}\bar{z} + Cz^0 + Dz^{-\frac{1}{2}}\ln(z) + Ez^{-\frac{3}{2}}\bar{z}\ln(z) \qquad (2)$$

This five parameter model, where the coefficients A and B are complex and the following assumptions are made $A = A_r + i3B_i$, $B = B_r + iB_i$, $D + E = 0$ can be solved in terms of stresses or displacements [2]. The work reported in this paper was obtained using digital image correlation (DIC) and the displacement solution is more useful:

$$\begin{aligned} 2\mu(u_x + iu_y) = {} & \kappa\left(-2(B_r + iB_i)z^{\frac{1}{2}} - 2Ez^{\frac{1}{2}}\ln(z) - \frac{C}{4}z\right) \\ & -z\left(-(B_r + iB_i + 2E)\bar{z}^{-\frac{1}{2}} - E\bar{z}^{-\frac{1}{2}}\overline{\ln(z)} - \frac{C}{4}\right) \\ & -\left((A_r + i3B_i)\bar{z}^{\frac{1}{2}} + D\bar{z}^{\frac{1}{2}}\overline{\ln(z)} - 2D\bar{z}^{\frac{1}{2}} + \frac{C}{2}\bar{z}\right) \end{aligned} \qquad (3)$$

Where u_x and u_y are the horizontal and vertical displacements respectively, $\mu = \frac{E}{2(1+\nu)}$; $\kappa = 3 - 4\nu$ (plane strain) or $\kappa = \frac{3-\nu}{1+\nu}$ (plane stress). u_x and u_y are shown explicitly below with the assumption $D + E = 0$.

$$\begin{aligned} 2\mu u_x = {} & r^{\frac{1}{2}}(-A_r - 2B_r\kappa - 2E)\cos\frac{\theta}{2} + r^{\frac{1}{2}}(2B_i\kappa - 3B_i)\sin\frac{\theta}{2} + r^{\frac{1}{2}}(B_r + 2E)\cos\frac{3\theta}{2} \\ & -r^{\frac{1}{2}}B_i\sin\frac{3\theta}{2} + r^{\frac{1}{2}}E\left[\ln(r)\left(\cos\frac{3\theta}{2} + (1-2\kappa)\cos\frac{\theta}{2}\right) + \theta\left(\sin\frac{3\theta}{2} + (1+2\kappa)\sin\frac{\theta}{2}\right)\right] \\ & -\frac{C}{4}r(1+\kappa)\cos\theta \end{aligned} \qquad (4)$$

$$\begin{aligned} 2\mu u_y = {} & r^{\frac{1}{2}}(-2B_i\kappa - 3B_i)\cos\frac{\theta}{2} + r^{\frac{1}{2}}(A_r - 2B_r\kappa + 2E)\sin\frac{\theta}{2} + r^{\frac{1}{2}}(B_r + 2E)\sin\frac{3\theta}{2} \\ & +r^{\frac{1}{2}}B_i\cos\frac{3\theta}{2} + r^{\frac{1}{2}}E\left[\ln(r)\left(\sin\frac{3\theta}{2} - (1+2\kappa)\sin\frac{\theta}{2}\right) - \theta\left(\cos\frac{3\theta}{2} + (1+2\kappa)\cos\frac{\theta}{2}\right)\right] \\ & +\frac{C}{4}r(3-\kappa)\sin\theta \end{aligned} \qquad (5)$$

CRACK GROWTH RATE TESTING

Compact tension (CT) specimens were machined from 2mm thick 2024-T6 aluminium CT specimen with non-standard dimensions [2]. A jeweller's saw with blade thickness of 0.15 mm was used to extend the notch tip into slits some 5 mm long at angles of 30°, 45° or 60° to the original horizontal notch plane; Fig. 1 shows typical CT specimens used in this work. A fatigue crack some 2 mm long was then grown in Mode I collinear with this slit. This was achieved by starting with a larger dimension CT specimen with additional loading holes in a similar fashion to the disk-shaped compact specimen described by Ding et al [4]. The specimen was then machined to final dimensions and the inclined fatigue crack extended under vertical uniaxial loading, giving a combination of mixed Mode I and Mode II crack tip stresses. The applied load ratio was R = 0.1 and the peak load was 1.2 kN. A Dantec digital image correlation (DIC) system operating in 2D mode was used to measure the crack tip displacement field and to compare the predictions of the CJP model with the measured displacement field data. A facet size of 17 pixels with a centre-line pitch of 17 pixels was used with a magnification of 107 pixels per millimetre.

Digital image correlation requires a fine speckle pattern to be sprayed onto the side of the specimen on which displacement is being measured. Increments in crack length during a period of fatigue cycling were monitored using acetate surface replicas taken on the opposite side of the specimen, which was polished to improve the visibility of the crack. The acetate replicas could then be inspected at magnifications up to 1,000x using an optical microscope and the crack increments accurately measured. In calculating da/dN, the total increment in crack length was used, taken account of any local deviations in path during the incremental growth.

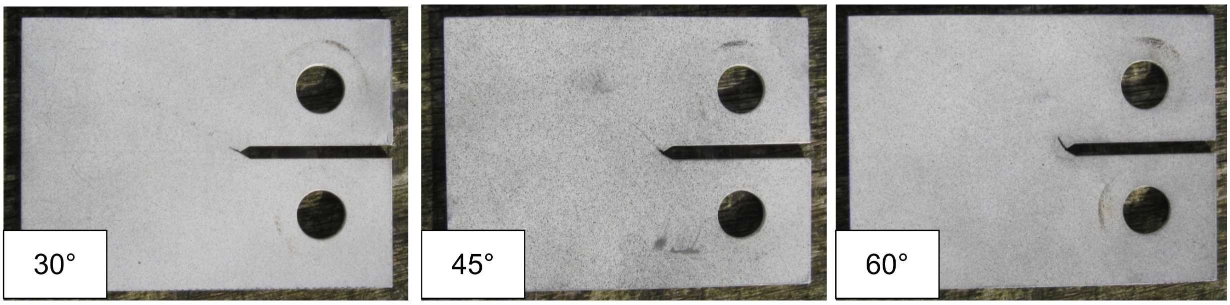

Figure 1: Typical aluminium CT specimens used in this work. In these specimens the initial inclined slit has been extended by fatigue crack growth.

Once a crack of sufficient size had been grown under biaxial stress conditions, sequential series of images were acquired from which the K_{min} and K_{max} values could be obtained after incremental amounts of crack growth. The experimental procedure followed in acquiring these images for analysis was the same for every specimen and is outlined in Fig. 2.

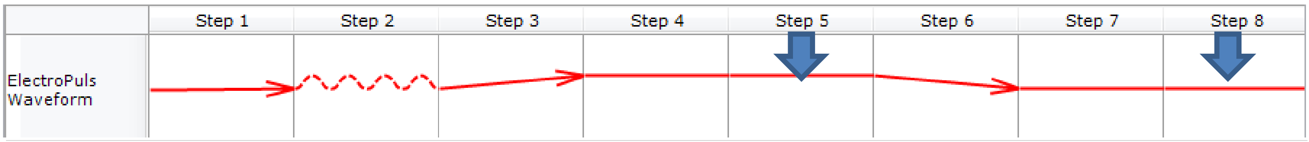

Figure 2: Illustration of the test procedure followed during the digital image correlation procedure.

Step 1	the P_{min} load of 150 N load is applied;
Step 2	load cycling of the specimen between P_{max} = 1,500 N and P_{min} = 150 N;
Step 3	the specimen is ramped up to the maximum load of 1,500 N;
Step 4	maximum load was held for a period of time (30s in this instance) whilst
Step 5	a signal is sent to the Dantec system to take an image;
Step 6	the load was decreased to the P_{min} value of 150 N;
Step 7	the minimum load was held for a period of time (30s in this instance) whilst
Step 8	a signal is sent to the Dantec system to take an image;

Finally, a surface replica was taken to measure the crack length increment during the applied number of cycles.
The complexities in this process due to the change in crack angle with growth can be illustrated through the crack path shown in Fig. 3 for the SP5A specimen that contained an initial 45° slit. In each case the angle is measured from the load line and the angle was then used to rotate the data for input into the CJP model.

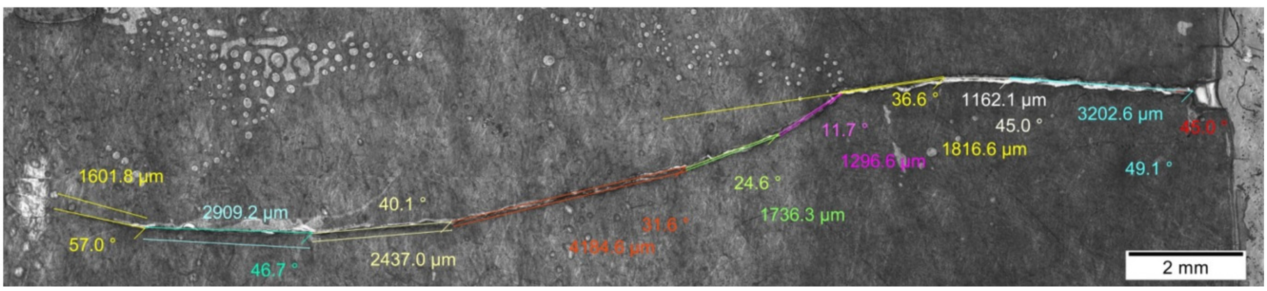

Figure 3: Illustration of the varying crack path angle on an acetate surface replica for specimen SP5A with an initial 45° slit. The arrow indicates the direction of the applied load during the crack growth under biaxial stresses.

RESULTS

The biaxial CJP model provides values for K_F, K_R, K_S, K_{II} and the T-stress. The values determined for K_F and K_R are shown in Fig. 4 and 5.

Several points can be observed from this data; firstly, in the CJP model negative stress intensity values of K_R and K_S can occur and this is reasonable and sensible for stress intensity factors that may act to either retard or accelerate crack advance. Secondly, there is considerable scatter between stress intensity factors obtained with nominally similar initial slit angles; part of this is likely to arise from the rather large variations in actual crack angle that occur during fatigue crack growth under mixed mode loading conditions, while another influence may be that fact that crack length is being measured on only a single surface. This later problem is not easy to resolve as potential drop techniques will also be subject to influence by crack angle variation. It was also the case that the crack growth data was subject to considerable point-to-point scatter and in further analysis of the results a 3-point sliding average technique was used to smooth the growth rate data. This approach is believed to be justified as many of the lower growth rate data points are bounded by much higher growth rates immediately before and after the lower value, but it does lead to a lower overall range of crack growth rate.

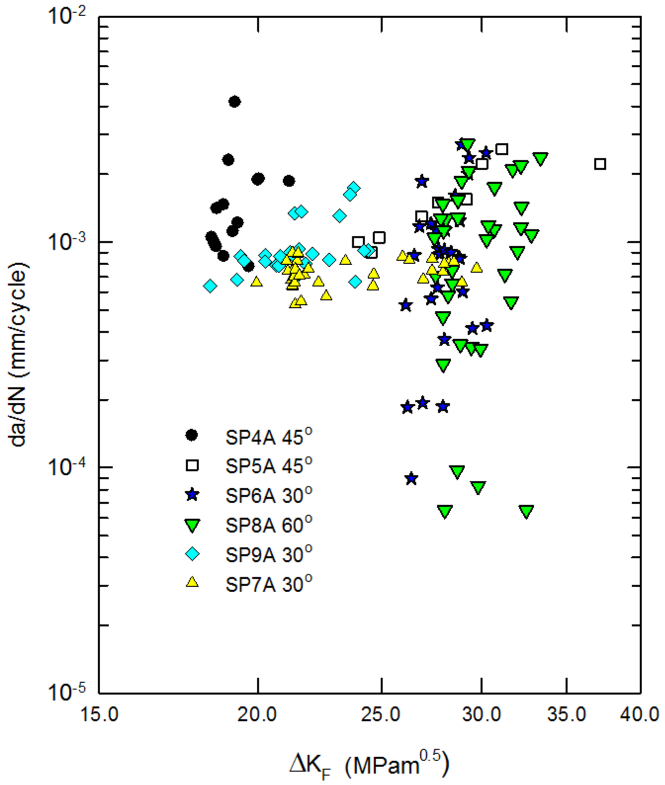

Figure 4: Fatigue crack growth rate data for all inclined crack specimens plotted against K_F.

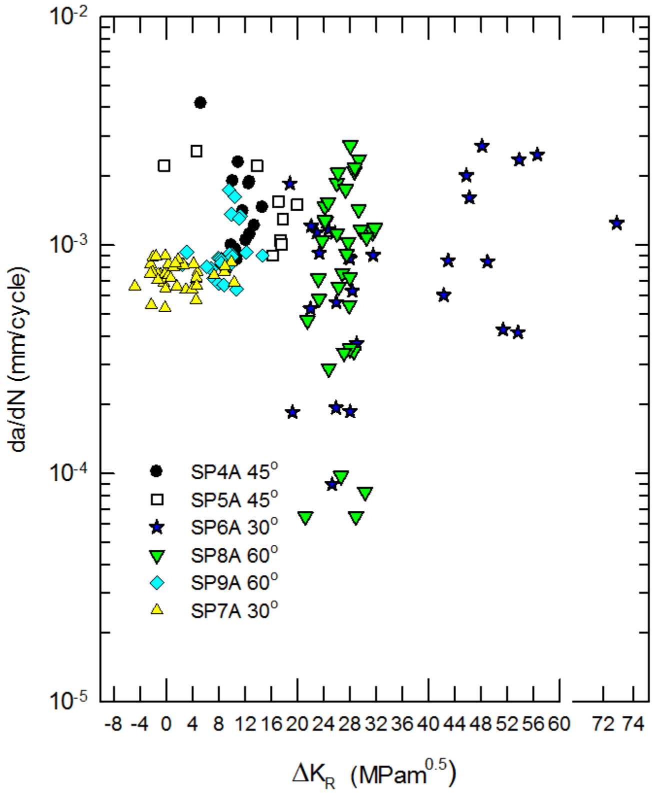

Figure 5: Fatigue crack growth rate data for all inclined crack specimens plotted against K_R.

A significant reduction in scatter can be achieved by considering the vector sum of the two stress intensity factors that the CJP model assumes are driving crack growth, K_F and K_{II} and the vector sum of the two retarding stress intensity factors K_R and K_S, and then finding the vector sum of the net driving force as shown in Eq. (6):

$$\text{Net driving force} = \sqrt{\sqrt{\left(K_F^2 + K_{II}^2\right)}^2 - \sqrt{\left(K_R^2 + K_S^2\right)}^2} \tag{6}$$

The data obtained by using this equation is shown in Fig. 6 and is difficult to interpret in classical da/dN versus stress intensity factor terms, as while the resulting curve is reasonably bilinear for the data obtained thus far in the work, it appears to indicate an almost constant growth rate for specimens with slits at initial angles of 30° and 60° over the complete range of net stress intensity factor considered in this work. Only the data for the two specimens with initial 45°

slits show a trend of increasing growth rate with increasing net driving force. The correlation between the 30° and 60° crack growth rate data is reassuring in the sense that, for a crack growing with a biaxial crack tip stress state under uniaxial loading, one would expect that the vector sum driving force in the two cases would be the same and hence deliver the same growth rate at equivalent values. However, this does not explain the reason underlying the approximately constant growth rate observed.

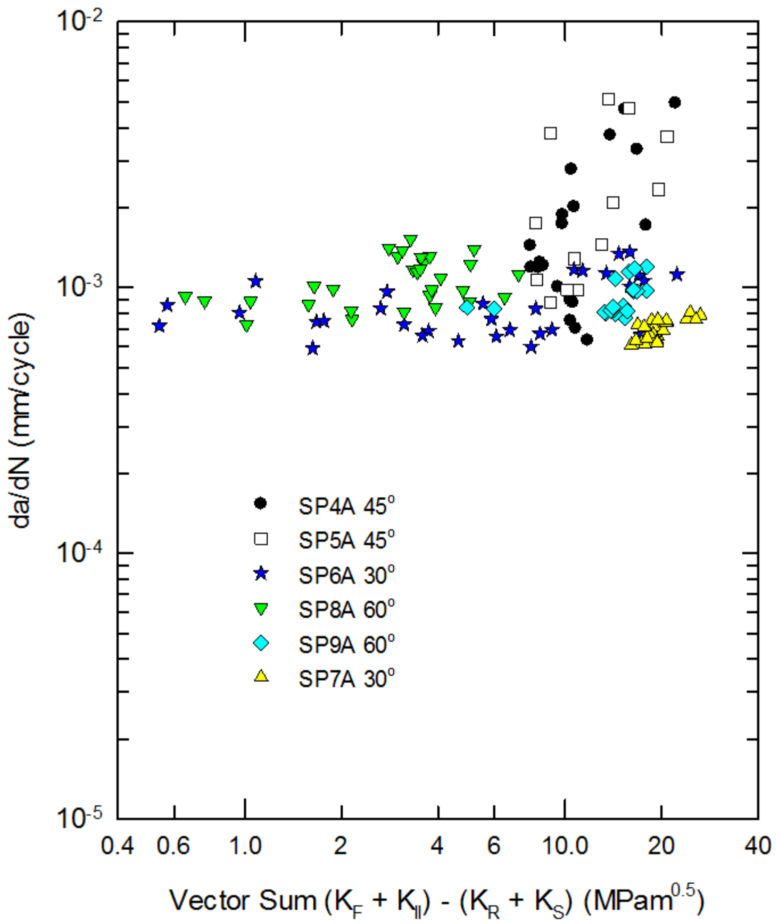

Figure 6: Reduction in scatter obtained by using the net vector sum driving force, expressed as a stress intensity factor.

CONCLUSIONS

The work completed since the Malaga Crack Tip Fields conference in 2013 has demonstrated that the CJP model is better able to characterise experimentally determined crack tip plastic zone size and shape that the original Williams model for crack tip stresses, irrespective of whether two terms or five terms are used in the Williams expansion [1]. Work on characterising the growth of inclined cracks in compact tension specimens under uniaxial loading (which therefore experience biaxial crack tip stress fields), using stress intensity parameters obtained from the CJP model, has been successful in achieving a sensible rationalisation of crack growth rate data, although some observations are not easily explained. Nonetheless, considering the complexity of characterising crack growth rates for cracks with an initial orientation of 30°, 45° or 60° to the horizontal and which subsequently change angle during growth, the results found so far indicate that there is value in further pursuing the CJP approach.

The present authors are taking several approaches to further refinement of the CJP model, by considering whether an energy-based approach to crack tip fields would be more useful for complex crack path situations, and by exploring the mathematical solution for an interfacial stresses around a plastic inclusion embedded in an elastic body. Alongside this work, further crack growth data will be acquired from CT specimens with horizontal cracks while the size and shape of plastic zones around growing fatigue cracks remains under active consideration.

REFERENCES

[1] James, M.N., Christopher, C.J., Lu, Yanwei, Patterson, E.A., Local crack plasticity and its influences on the global elastic stress field, Int. J. Fatigue, 46 (2013) 4-15.
[2] Christopher, C.J., Laboviciute, C., James, M.N., Patterson, E.A., Extension of the CJP model to mixed mode I and mode II, Frattura ed Integrita Strutturale, 7 (2013) 161-166.
[3] Tada, H., Paris, P.C., Secondary elastic crack tip stresses which may influence very slow fatigue crack growth—additional results, Int. J. Fatigue, 27 (2005) 1307–1313.
[4] Ding, F, Zhao, T., Jiang, Y., A study of fatigue crack growth with changing load direction, Engng Fract. Mech., 74 (2007) 2014-2029.