A theoretical model for predicting the Peak Cutting Force of conical picks

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ABSTRACT. In order to predict the PCF (Peak Cutting Force) of conical pick in rock cutting process, a theoretical model is established based on elastic fracture mechanics theory. The vertical fracture model of rock cutting fragment is also established based on the maximum tensile criterion. The relation between vertical fracture angle and associated parameters (cutting parameter η and ratio B of rock compressive strength to tensile strength) is obtained by numerical analysis method and polynomial regression method, and the correctness of rock vertical fracture model is verified through experiments. Linear regression coefficient between the PCF of prediction and experiments is 0.81, and significance level less than 0.05 shows that the model for predicting the PCF is correct and reliable. A comparative analysis between the PCF obtained from this model and Evans model reveals that the result of this prediction model is more reliable and accurate. The results of this work could provide some guidance for studying the rock cutting theory of conical pick and designing the cutting mechanism.

KEYWORDS. Conical pick; Peak Cutting Force; Fracture angle.

INTRODUCTION

In tunneling and mining, interaction between cutting mechanism (shearer drum and cutting head) and rock realizes rock fragmentation. Cutting ability, crushing effect and production efficiency of cutting mechanism, determine the working performance of tunneling and mining equipment. However, the PCF prediction of conical pick plays an important role in the design of cutting mechanism. A considerable amount of research has been conducted on the PCF prediction base on theoretical, experimental and numerical method. The first PCF prediction model of conical pick was established by Evans based on the maximal tension stress theory [1], and Roxborough & Liu [2] and Goktan [3] appropriately modified the Evans mathematical model. According to experiment data from various rock cutting by conical pick, regression expressions between cutting force and rock compressive strength, tensile strength, dynamic and static modulus of elasticity, brittle index were established by Bilgin [4-5]. Tiryaki [6] adopted multiple linear and non-linear regression, regression tree model and neural networks method to predict the PCF of conical pick. Numerical method [7-9] is also applied to predict the PCF of conical pick. Viewing the references mentioned above, we find that: Compared with experimental date, the result calculated by current theoretical model exists a notable divergence; For the limitation of experimental date, application of empirical models is limited to a specific scope; Experimental method and numerical simulation method can obtain proper cutting force, but some disadvantages exist such as high cost and low efficiency. For these reasons, the vertical fracture mechanics model of rock cutting fragment is established based on the maximum tensile criterion and rock cutting theory in this paper firstly; then, the theoretical model for predicting the PCF of conical pick is established based on elastic fracture mechanics theory; finally, the reliability and accuracy of PCF model is verified with experimental date.

THEORETICAL MODEL

The problem of the indentation of the plane surface of elastic solid with a rigid body was first considered by Boussinesq [10], then Sneddon [11] adopted hankel transforms and elementary solution to solve the Boussinesq problem, and the total penetration depth and force of the rigid body were presented. Therefore, the applied force of conical pick on rock could be approximately expressed as:

$$P = \frac{2Eh^2 \tan\alpha}{\pi\left(1-v^2\right)} \tag{1}$$

where:
P is the penetration force of conical pick;
E is elastic modulus of rock;
α is semi-angel of conical pick;
v is poisson ratio of rock;
h is the penetration depth of conical pick.
Integrating Eq.1 with respect to the penetration depth, the work W_E can be obtained and it is expressed as:

$$W_E = \int_0^h P\,dh = \int_0^h \frac{E\tan\alpha}{\pi\left(1-v^2\right)} h^2\,dh = \frac{E\tan\alpha}{3\pi\left(1-v^2\right)} h^3 \tag{2}$$

Therefore, when the main rock fragment formed, the total work W_T can be expressed as:

$$W_T = \frac{2E\tan\alpha}{3\pi\left(1-v^2\right)} h_{\max}^3 \tag{3}$$

where $h_{\max}$ is the maximum penetration depth at the time of main rock fragment formed.
The fracture surface of main rock fragment is simplified without influence of PCF prediction, and it is shown in Fig.1. As Fig.1 shown, the new fracture surface area A of main rock fragment can be expressed as:

$$A = \frac{ef}{\sin\theta} = \frac{d^2\tan\psi\tan\theta}{\sin\theta} = \frac{d^2\tan\psi}{\cos\theta} \tag{4}$$

where:
e and f are the geometry shape parameters of main rock fragment;
d is cutting depth of conical pick;
θ is horizontal fracture angles;
ψ is vertical fracture angle.

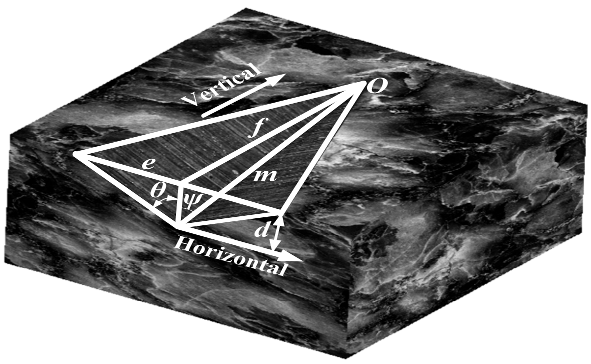

Figure 1: Schematic diagram of fracture surface of rock fragment.

According to the relevant references, the work done by conical pick mainly translate into rock fracture energy, damage energy, plastic strain energy and so on [12-13]. The fracture energy E_F can be express as:

$$E_F = kW_T \tag{5}$$

where k is an ideal coefficient which only has relation with pick's shape and cutting angle, and it can be obtained through rock cutting test. In *Verification of the Model* paragraph, acquisition process of k value will be presented.

On the basis of Griffith fracture mechanics theory [14], the fracture energy E_F for generating new fracture surface when the main rock fragment formed can be expressed as:

$$E_{\mathrm{F}} = 2G_{\mathrm{s}}A = \frac{2G_{\mathrm{s}}d^2\tan\psi}{\cos\theta} = \frac{2K_{\mathrm{I}}{}^2 d^2\tan\psi}{E\cos\theta} \tag{6}$$

where G_s represents surface free energy per unit area of rock material; K_I stands for fracture toughness of rock material in the type of model I cracking, it can be approximately calculated by $\sigma_t/6.88$ [15]; σ_t is rock tensile strength.

Combining Eqs. 3-6, the maximum penetration depth of conical pick before main rock fragment formed can be expressed by:

$$h_{\max} = \left(\frac{3\pi K_I^2\, d^2\left(1-\nu^2\right)\tan\psi}{k\,E^2\,\tan\alpha\,\cos\theta}\right)^{\frac{1}{3}} \tag{7}$$

Submitted Eq.7 to Eq.1, then the peak cutting force of conical pick P_c can be expressed as:

$$P_c = 2\left(\frac{\tan\alpha}{\pi E(1-\nu^2)}\right)^{\frac{1}{3}}\left(\frac{3K_{\mathrm{I}}^2\tan\psi}{k\cos\theta}\right)^{\frac{2}{3}} d^{\frac{4}{3}} \tag{8}$$

CALCULATION OF ROCK FRACTURE SURFACE AREA

The similarity of geometry shape of brittle material (glass, ceramic, rock, coal and so on) fragment under indenters or cutting tools has been verified through experiments, and there are linear relationship between width, length of fragments and cutting depth [16-18]. According to Eq.8, it is visible that the area calculation of new facture surface generated by conical pick is the basis for predicting the peak cutting force of conical pick, which means to establish the calculative method for horizontal and vertical fracture angle of rock fragment.

CALCULATION OF HORIZONTAL FRACTURE ANGLE

Horizontal fracture angle of rock fragment is directly related to the optimal intercept of conical picks. The optimal intercept of conical picks in rock cutting process is 3.46 times the cutting depth according to Evans calculate method [1]. Optimal intercepts of 22 different rock types have been obtained through rock cutting experiments by Bilgin [5], and their values are very close to 3.23 times the cutting depth. In view of 7% difference between Evans calculated result and Bilgin experimental result, Evans calculate method is considered reliable and correct. So, in this paper, horizontal fracture angle of rock fragment in the mathematical model for predicting the peak cutting force will adopts Evans calculate model, which means that horizontal fracture angle is equal to 60 degree.

CALCULATION OF VERTICAL FRACTURE ANGLE

The influence of cutting angle on the rock fragment is ignored in Evans theory, but the geometric shape of rock cutting fragment has play an important role in perfect prediction of peak cutting force. Base on Evans theory, some assumptions are put forward in present paper: rock broken is caused by tensile failure, and it accords with the maximum tension theory; the generated total force by tensile stress in fracture surface is through the center of arc AC; the crushing zone caused by conical pick head is shown in Fig.2, and AG is an arc with the center O.

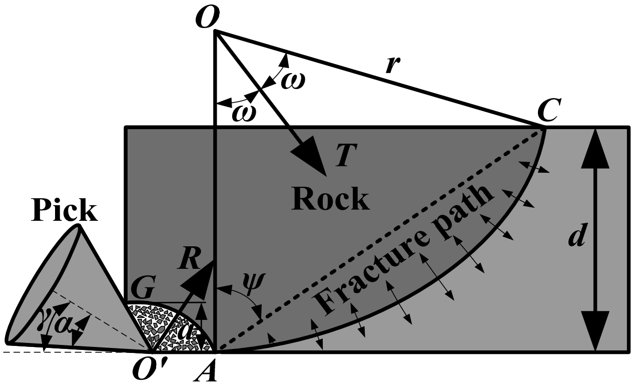

Figure 2: Vertical mechanical model of rock fragment.

Based on these assumptions, vertical mechanical model of rock fragment by conical pick is shown in Fig.2. According to the geometrical relationship in Fig.2, the force *R* is the resultant force of extrusion forces which acted on crush zone, and it has the relation with rock compressive strength and crush area. Meanwhile, the force *R* also increased with the height of crush zone (*a*), and it will reach a maximum when the torques caused by force *T* and force *R* on point C get balance. The force *R* can be expressed as:

$$\mathrm{R} = \sigma_c \int_{-(90-\eta)}^{90-\eta} \frac{a}{\sin(2\eta)} \cos\lambda d\lambda = \frac{\sigma_c a}{\sin\eta} \quad (9)$$

where:
γ is the cutting angle of conical pick;
a is the height of crushing zone;
σ_c represents rock comprehensive strength;
η is the cutting parameter of pick, and it equals to $(\gamma+\alpha)/2$.
Rock will fracture along the arc AC when the force of conical pick applied on rock is big enough. However, the extreme state of rock fragment is determined by rock tensile strength, and the force *T* is the resultant force of tensile forces which acted on crack path. It can be expressed as:

$$T = \sigma_t r \int_{-\omega}^{\omega} \cos\beta d\beta = 2\sigma_t r \sin\omega = \frac{\sigma_t d}{\sin\omega} \quad (10)$$

where:
r is the radius of the fracture arc AC;
ω stands for the complementary angle of vertical fracture angle.
The force *R* and force *T* will reach a moment balance when the rock fragment forms. Therefore, an equation between them can be obtained according to geometrical conditions, and it can expressed as:

$$\mathrm{R}\left[a\sin\eta + \frac{d}{\sin\omega}\cos(\eta+\omega)\right] - T\frac{d}{2\sin\omega} = 0 \quad (11)$$

Submitting Eq.9 and Eq.10 to Eq.11, we have:

$$\frac{\sigma_c a}{\sin\eta}\left[a\sin\eta + \frac{d\cos(\eta+\omega)}{\sin\omega}\right] - \frac{\sigma_t d^2}{2\sin^2\omega} = 0 \quad (12)$$

Changing Eq.12 to the form of quadratic equation:

$$(\frac{a}{d})^2 + \frac{\cos(\eta+w)}{\sin\eta\sin\omega}(\frac{a}{d}) - \frac{1}{2\sin^2\omega}\frac{\sigma_t}{\sigma_c} = 0 \quad (13)$$

Solving Eq.13, the parameter a/d can be obtained and expressed as:

$$\frac{a}{d}=\frac{\sqrt{\frac{\cos^2(\eta+\omega)}{4\sin^2(\eta)}+\frac{\sigma_t}{2\sigma_c}}-\frac{\cos(\eta+\omega)}{2\sin\eta}}{\sin\omega} \quad (14)$$

In this paper, the derivation process of theoretical model takes Evans work as reference. So, the energy of cutting system will be minimum at the moment of rock broken presumed by Evans as an equilibrium situation. For this reason, this paper can use the 'minimum energy theory' as theoretical basis. According to the minimum energy theory:

$$\mathrm{d}(a/d)/\mathrm{d}\omega=0 \quad (15)$$

Now, the relationship between η and ω can be obtained and expressed as:

$$\frac{\frac{\sin(\eta+\omega)}{2\sin\eta}-\frac{\cos(\eta+\omega)\sin(\eta+\omega)}{4\sin^2(\eta+\omega)\sqrt{\frac{\sigma_t}{2\sigma_c}+\frac{\cos^2(\eta+\omega)}{4\sin^2\eta}}}}{\sin\omega}-\frac{\cos\omega(\sqrt{\frac{\sigma_t}{2\sigma_c}+\frac{\cos^2(\eta+\omega)}{4\sin^2\eta}}-\frac{\cos(\eta+\omega)}{2\sin\eta})}{\sin^2\omega}=0 \quad (16)$$

B represents the ratio of rock compressive strength to tensile strength. From Eq.16, we can conclude that it is difficult to get the analytical solution of Eq.16. Consequently, the effects of cutting parameter η and ratio B on vertical fracture angle ψ are investigated, and the change law of ψ along with η and B is shown in Fig.3. Fig.3 shows that ψ increases with η and B. In order to get intuitive relationship among ψ, η and B, the data points of numerical solution in Fig.3 are regressed based on space surface regression method. The regression formula of vertical fracture angle ψ can be expressed as Eq.17, and the correlation coefficient of formula is 0.994.

$$\psi=48.87+0.526B+0.224\eta \quad R^2=0.994 \quad (17)$$

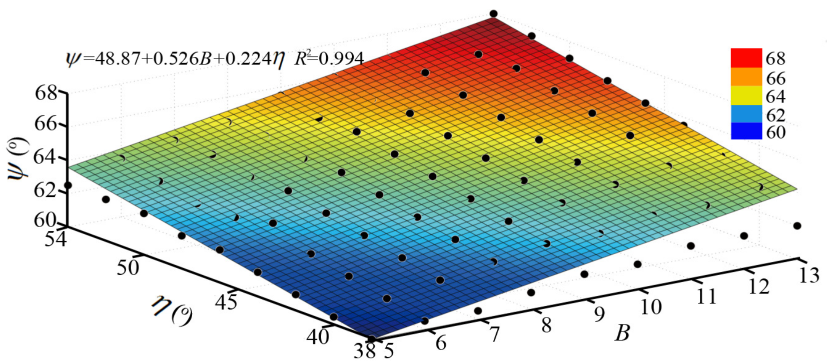

Figure 3: Variation law of vertical fracture angle of rock fragment.

VERIFICATION OF THE MODEL

Horizontal fracture angle of rock fragment is directly related to the optimal intercept of conical picks. The optimal intercept of conical picks in rock cutting process is 3.46 times the cutting depth according to Evans calculate method [1]. Optimal intercepts of 22 different rock types have been obtained through rock cutting experiments by Bilgin [5], and their values are very close to 3.23 times the cutting depth. In view of 7% difference between Evans calculated result and Bilgin experimental result, Evans calculate method is considered reliable and correct. So, in this paper, horizontal fracture angle of rock fragment in the mathematical model for predicting the peak cutting force will adopts Evans calculate model, which means that horizontal fracture angle is equal to 60 degree.

Verification of vertical fracture angle

In order to verify the correctness of vertical mechanical model of rock fragment in this paper, rock cutting experiments are carried in laboratory as shown in Fig.4. The semi-angel of conical pick is 40 degree, cutting angle of conical pick is selected in range of 45~55 degree according to actual working condition, and the mechanical properties of marble is shown in Tab.1. The shape and interrelated geometric parameters are shown in Fig.5. Fig.6 shows the theoretical and experimental vertical fracture angle under different cutting angle, the difference less than 5 percent between them indicates that the established vertical mechanical model of rock fragment is valid.

Properties	Value
Modulus of elasticity E(GPa)	19
Density ρ(kg/m^3)	2650
Fracture toughness K_I (MPa·m$^{1/2}$)	1.1
Compression strength σ_c(MPa)	103.2
Tensile strength σ_t(MPa)	7.1

Table 1: Mechanical properties of marble.

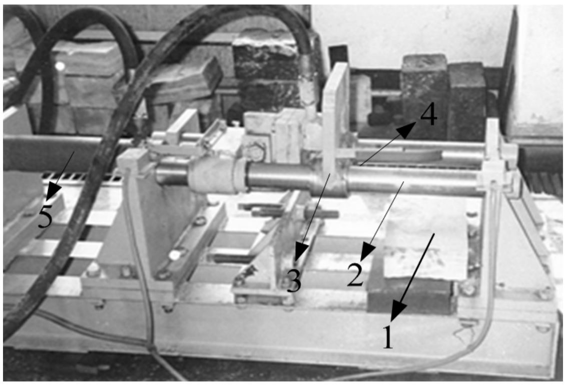

Figure 4: Laboratory furniture of rock cutting linearly: 1-rock specimen; 2-guideways; 3-clamping device of conical pick; 4-conical pick; 5- advancing hydro-cylinder.

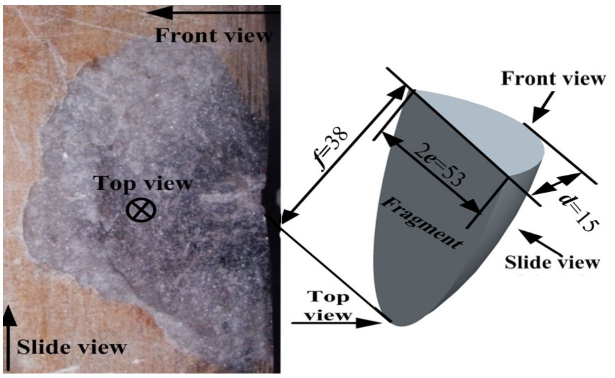

Figure 5: Geometric feature parameters of rock fragment.

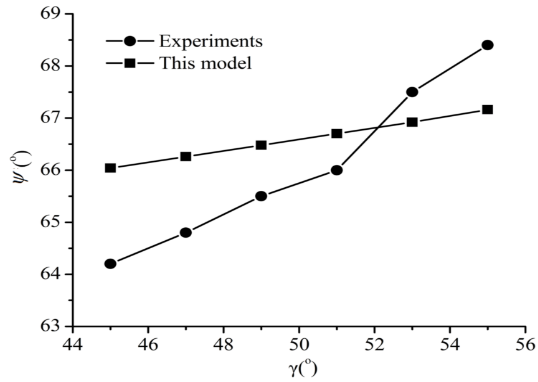

Figure 6: Vertical fracture angle of experiments and theoretical model.

Verification of peak cutting force

The k is a geometric factor of the pick independent of material properties which has been mentioned in front contents. In reference [18], the researchers also took an intensive study and detailed explanation of the k. According to Eq.5, the k can be obtained by experiments. Fig.7 is the variation curve of cutting force in rock cutting process, which corresponds to Fig.5. So, the k can be expressed as:

$$k = \frac{E_F}{W_T} = \frac{2K_I^2 d^2 \tan\psi}{E\cos\theta \int P(h)\mathrm{d}h} \approx \frac{4K_I^2 d^2 \tan\psi}{E\cos\theta P_c h_{max}} \tag{18}$$

Submitted the rock mechanical properties and the geometric parameters of rock fragment to Eq.18, then the k can be obtained as 0.0102. According to the value of k, we can conclude that the fracture energy for generating new fracture surface accounts for only a small part of total work, and the most part of total work is used for rock plastic deformation, rock damage, crushing zone formed and so on. Now, we get the value of k, and if we also get the other parameter values, we can obtain the peak cutting force P_c by Eq.8.

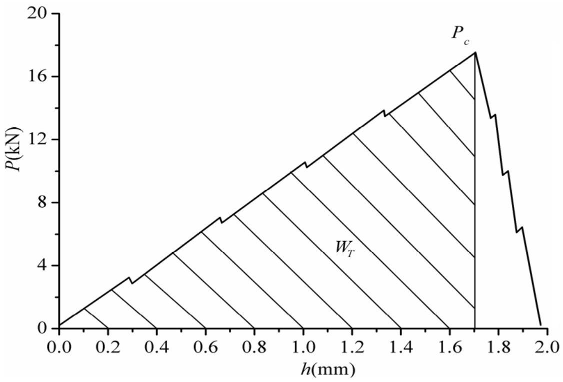

Figure 7: Variation curve of cutting force.

The peak cutting forces of experiments, this model and Evans model with different rock types and cutting parameters are shown in Tab.2 [5-7]. The relationships between experimental peak cutting force with theoretical peak cutting force from this model and Evans model are investigated by linear regression method, and their linear regression results are shown in Tab.3. The significance of regression result less than 0.05 indicates that the regression relationships are correct and reliable. Fig.8 and Fig.9 shows the fitted relationships between experimental peak cutting force and two theoretical models (Evans model and this model) respectively. The correlation between experimental and theoretical peak cutting force of this model is better than Evans's, and the slopes of the fitted line equations are 2.11 and 5.13. It indicates that the prediction of peak cutting force by this model has more correctness and reliabilities than Evans theory.

Type	σ_c	σ_t	E	K_I	d=9mm			d=5mm		
					P_c^{Exp}(N)	P_c(N)	P_c^E(N)	P_c^{Exp}(N)	P_c(N)	P_c^E(N)
Chromite1	32	3.7	3.5	0.538	14830	8230	3660	7160	3759	920
Chromite2	47	4.5	2.3	0.654	26490	12642	3690	10210	5773	920
Chromite3	46	3.7	2.9	0.538	16240	9309	2550	8710	4251	3190
Harsburgite	58	5.5	2.1	0.799	26910	17057	4470	14970	7789	1120
Serpantinite	38	5.7	2.3	0.828	20150	16338	7320	7850	7462	1830
Trona	30	2.2	3.4	0.320	12260	4503	1380	3880	2056	350
Anhydrite	82	5.5	11.0	0.799	16300	10558	3160	12520	4822	790
Sandstone1	114	6.6	17.0	0.959	25920	12138	3270	19690	5544	820
Sandstone2	174	11.6	28.0	1.686	48100	20946	6620	23250	9566	1660
Sandstone3	87	8.3	33.3	1.206	15920	11739	6780	9090	5361	1700
Tuff1	10	0.9	1.1	0.131	4020	1911	690	2050	873	170
Tuff2	11	1.2	1.4	0.174	11840	2509	1120	7080	1145	280
Tuff3	27	2.6	2.4	0.378	7200	5993	2140	3770	2735	540
Tuff4	14	1.5	1.6	0.218	7300	3239	1380	2830	1479	340
Tuff5	19	2.3	1.3	0.334	7350	6037	2380	3440	2757	600
Tuff6	6	0.2	0.4	0.029	2180	523	57	1330	238	14

P_c^{Exp}: The PCF of experiments; P_c^E: The PCF of Evans model; P_c: The PCF of this model

Table 2: Rock mechanical property and the PCF.

	Item	DF	SS	MS	F-value	Prob>F
P_c^{exp}- P_c	Model	1	2.5e9	2.5e9	133.7	1.4e-12
	Error	30	5.6e8	1.9e7		
	Total	31	3.1e9	1.6		
P_c^{Exp}-P_c^E	Model	1	1.6e9	1.6e9	31.7	3.7e-6
	Error	30	1.5e9	5.0e7		
	Total	31	3.1e9			
DF: Degrees of freedom; SS: Sum of squares; MS: Mean square						

Table 3: The regression analysis results of the PCF obtained from different method.

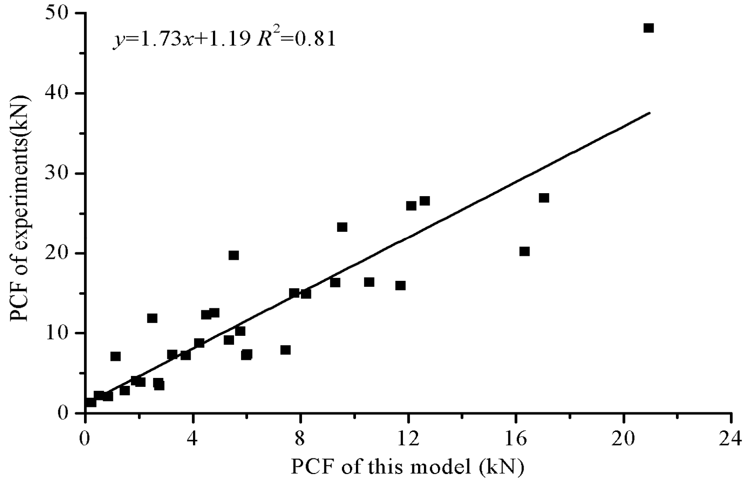

Figure 8: The relationship between the predicted and experimental PCF.

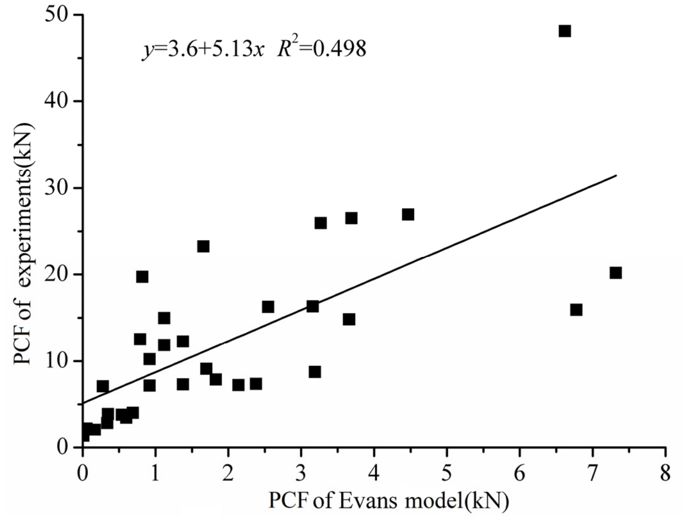

Figure 9: The relationship between the PCF of Evans model and experiments.

CONCLUSIONS

In this paper, a theoretical model for predicting the peak cutting force has been set up and verified, we get following conclusions:

(1) The theoretical model of rock vertical fracture has been established by maximum tensile criterion and verified

through experiments. The regression formula of vertical fracture angle also has been obtained by numerical analysis and polynomial regression. All of these work can provide a basis for the research on rock cutting theory.
(2) A theoretical model for predicting peak cutting force of conical pick in rock cutting process has been established by elastic fracture mechanics theory. The regression analysis between the results of experimental and predicted by this model shows that correlation coefficient is equal to 0.81 and significance is less than 0.05.
(3) The relationships between experimental peak cutting force and calculated values by the present model and Evans model are investigated by linear regression analysis, and it shows that the prediction of peak cutting force of this model has more correctness and reliabilities than Evans model. The model established by this paper can provide a better guidance for design and study of conical pick and cutting mechanism.

Acknowledgements

The paper was financially supported by the National 863 Plan of China(2012AA062104), National Natural Science Foundation of China(51005232), Jiangsu Ordinary University Graduate Students Scientific Research Innovation Project(CXZZ11-0289), the Jiangsu Provincial Natural Science Foundation of China (No. BK20131116), the Fundamental Research Funds for the Central Universities (Project No.2012QNA22) and the Priority Academic Program Development of Jiangsu Higher Education Institutions.

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